O'QUV QO'LLANMA

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HISOBLASH USULLARI

"O'ZKITOBSAVDONASHRIYOTI" NMIU

O'ZBEKISTON RESPUBLIKASI OLIY TA'LIM,
FAN VA INNOVATSIYALAR VAZIRLIGI

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HISOBLASH USULLARI

(o'quv qo'llanma)

O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar vazirligi tomonidan (51302100-amaliy matematika va informatika) bakalavriyat ta'lim yo'nalishi talabalari uchun

TOSHKENT
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Annotasiya

Oquv qo'llanma universitetlarning 5130200-«Amaliy matematika va informatika» bakalavr yo'nalishi uchun mo'ljallangan. Unda sonli tenglamalar, ularning chiziqli va nochiziq sistemalari, xos sonlar muammosi, interpolyasiya, taqribiy integrallash, oddiy differensial tenglamalar uchun boshlang'ich va chegara masalalar, xususiy hosilali differensial tenglamalar uchun chegara masalalar, integral tenglamalarning taqribiy yechish usullari qaralgan. Barcha usullarning Mathcad va Maple tizimlardagi yiriklashtirilgan algoritmlari, programmalari, yechimlarning grafik tasvirlari keltirilgan.

Аннотация

Учебное пособие предназначен для студентов направления 5130200-«Прикладная математика и информатика». В нем содержится приближённое решение одного уравнения, систем линейных и нелинейных уравнений, проблема собственных значений, задачи интерполяции и приближённого интегрирования, задачи Коши и краевых задач для обыкновенных дифференциальных уравнений, дифференциальных уравнений с частными производными и интегральных уравнений, укрепнённые алгоритмы. Во всех решениях данны программы и графики всех методов в математических системах Mathcad ва Maple.

Annotation

The textbook is intended for students of the direction 5130200 - "Applied Mathematics and Informatics". It contains an approximate solution of a single equation, systems of linear and non-linear equations, an eigenvalue problem, interpolation and approximate integration problems, Cauchy problems and boundary value problems for ordinary differential equations, partial differential equations and integral equations, hardened algorithms. In all solutions, these programs and graphics of all methods in the mathematical systems Mathcad and Maple.

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KIRISH

O'zbekiston Respublikasi mustaqillikga erishgandan so'ng ijtimoiy-iqtisodiy, ilmiy-texnikaviy, madaniy va ma'naviy hayotda ayniqsa ta'lim sohasida o'z aksini topmoqda.

O'zbekistonda oliy ta'lim tizimini takomillashtirish, ta'lim sifatini jahon andozalari darajasiga yetkazish ta'lim tizimining eng dolzarb vazifasiga aylandi. Bu esa, barcha mutaxassisliklarni tayyorlash sifatini oshirishni taqozo etadi.

O'quv qo'llanmada keltirilgan ma'lumotlarni tushunish uchun talabadan algebra va analizdan dastlabki tushunchalar bilan tanish bo'lish talab etiladi.

Hisoblash usullari fani algebra va analiz masalalarini, differensial va integral tenglamalarni taqribiy yechishga bag'ishlangan. Bu yerda bitta chiziqsiz tenglama, ularning sistemalari, chiziqli tenglamalar sistemalari, xos vektorlar muammosi, interpolyasiya, taqribiy integrallash, oddiy va xususiy hosilali differensial va integral tenglamalarni taqribiy yechish masalalari, barcha usullarning Mathcad, Maple matematik tizimlaridagi yiriklashtirilgan algoritmlari, hisoblash eksperimentlari, yechimlarning jadval va grafik tasvirlari keltirilgan.

Shuni aytib o'tish joizki, respuplikamizda Hisoblash usullari fanidan o'z vaqtida bir necha masalalar toplamlari chop etilgan bo'lib, ulardan Abduqodirov A., Fozilov F., Umrzoqov T[1]., Abduhamidov A., Xudoynazarov S[2] larni keltirish mumkin. Bu kitoblar chiqqan davrlarda komputerlar, matematik tizimlar endigina oliy ta'limga kirib kelayotgan edi.

Fanni o'qtishning maqsadi: talabalarda turli masalalarni taqribiy yechishda algoritmlarning sifatini va imkoniyatlarini tahlil qilish hamda algoritmlarni yarata bilish ko'nikmalarini hosil qilishi, berilgan masalaning turini aniqlay olish, yechish usullarini to'g'ri qo'llay bilish va ushbu usullarning turg'unligini aniqlay bilish, dasturlash tillarini qo'llagan holda shaxsiy kompyuterlarda masalalarni yecha olish kompetensiyalarini shakllantirishdan iborat.

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[1] AbduqodirovA., FozilovF., Umrzoqov T., Hisoblash matematikasi va dasturlash. T.: O'qituvchi, 1995.-256 b.,
[2] Abduhamidov A., Xudoynazarov S. Hisoblash usullaridan mashqlar va laboratoriya ishlari. T.: O'zbekiston,1995.-224 b.

Fanni o'qtishning vazifalari: kurs mobaynida funksiyalarni yaqinlashtirish, taqribiy differensiyalsh va interpolyasiyalash, algebraning sonli usullari, oddiy va xususiy hosilali differensial tenflamalarni taqribiy echish usullarini, sonli hisoblash natijalarini malakali ravishda tahlil qilish usullarini o'rgatishdan iborat.

Fan bo'yicha talabalarning bilim, ko'nikma va malakalariga quyidagi talablar qo'yiladi. Talaba:

- hisoblash usullari o'quv fanini o'zlashtirish jarayonida amalga oshiraladigan masalalar doirasida bakalavr xatoliklar va ularning manbai; chiziqli tenglamalar sistemasini yechishning aniq usullari; chiziqli tenglamalar sistemasini yechishning iterasion usullari; chiziqsiz tenglamalar va tenglamalar sistemasini taqribiy yechish usullari; matrisa xos son va xos vektorlarini taqribiy hisoblash usullari; integrallarni taqribiy hisoblash formulalarini keltirib chiqarish; oddiy differensial tenglamalarga qo'yilgan masalalarni sonli yechish usullari; xususiy hosilali differensial tenglamalarni sonli yechish usullari haqida tasavvurga ega bo'lishi;
- xatoliklarni hisoblashni; matrisa normalarini hisoblashni; chiziqli tenglamalar sistemasini iterasiya usuli bilan yechishni; funksiyani yaqinlashtirish usullarini; integralni taqribiy hisoblash formulalarini; oddiy differensial tenglamalarga qo'yilgan masalalarni taqribiy yechish usullari xatoliklari bahosini; xususiy hosilali differensial tenglamalarga qo'yilgan masalani taqribiy yechish usullarini xatoliklarini bilishi va ulardan foydalana olishi;
- hisoblash usullari xatoliklarini baholash; taqribiy yechish usullarini tanlash; masalaning aniq va taqribiy yechimlari orasidagi farqni baholash; masalani taqribiy yechish uchun biror dasturlash tilidan foydalanish ko'nikmalariga ega bo'lishi kerak.

Masalan, chiziqsiz tenglamalar, interpolyasiya masalasi, integrallarni taqribiy hisoblash uchun 4-5- ta usul taklif etilgan. Bu talabaga o'zini o'zi nazorat qilish imkoniyatini beradi. Har bir mavzu asosiy tushunchalar, formulalar, savollar, usullarning matematik tizimlardagi yiriklashtirilgan algoritmlari bilan taminlangan.

Chiziqsiz tenglamalar va ularning sistemalarini taqribiy yechish usullari, universal oddiy iterasiya va Nyuton iterasiya usullari qaralgan va ularning matematik tizimlardagi yiriklashtirilgan algoritmlari keltirilgan.

Chiziqli algebra masalalarini taqribiy yechish usullari Gauss, Gauss-Jordan, progonka, oddiy iterasiya va Zeydel, relaksasiya iterasiya usullari qaralgan. Matrisaning xos sonlarini topishning det erminantni bevosita hisoblash, Krilov, Levyerrye, interpolyasiya, LU, QR-usullar uchun MathCAD, Mapleda yiriklashtirilgan algoritmlari keltirilgan.

Funksiyalarning interpolyasiyalash Nyuton, Lagranj va splayn-interpolyasiya formulalari, qoldiq hadlari qaralgan va usullarning MathCAD, Mapleda yiriklashtirilgan algoritmlarlari va ular asosida olingan sonli natijalar, grafik tasvirlar keltirilgan.

Funksiyalarni taqribiy integrallash Nyuton-Kotes, trapesiya, to'g'ri to'rtburchaklar, Simpson kvadratura formulalari, qoldiqlari qaralgan. Barcha usullarning MathCAD, Mapledagi alohida va umumlashgan yiriklashtirilgan algoritmlari keltirilgan.

ODT uchun Koshi masalasi mavzusida esa, takomillashgan, prognoz-korreksiya Eyler, Simpson, Runge-Kutta usullari qaralgan. ODT uchun chegara masalalar uchun kollokasiya, EKK, Galyorkin-Rits, chekli-ayirmalar usullari qaralangan.

XHDT mavzusida PDT, GDT va EDT lar uchun oshkor va oshkormas, Krank-Nikolson ayirmali sxemalari, Fredgolm, Volterr integral tenglamalar uchun uzluksiz va diskret iterasiya, kvadraturalar usullari qaralgan. Barcha usullar uchun matematik tizimlarda yiriklashtirilgan algoritmlar tuzilgan va natijalar olingan.

Matematika bu tashqi dunyoning geometrik formalari va sonli munosabaatlari haqidagi fandir (Katta ensikliopediyadan).

Ushbu o'quv qo'llanma hozirgi zamon "Hisoblash usullari" kursining Respublikamiz universitetlarining amaliy matematika va informatika, matematika yo'nalishlari bo'yicha qabul qilingan davlat ta'lim standartlari.

malaka talablari o'quv dasturlari asosida yozilgan va algebra va analizning, differensial va integral tenglamalarning taqribiy yechish usullarini o'z ichiga oladi. Bundan tashqari o'quv qo'llanma mazkur kurs bo'yicha qo'shimcha mashg'ulotlar, talabalar bilan mustaqil individual ta'lim dasrlarini o'tkazishda foydalanish mumkin. Hisoblash usullari fan sifatida bevosita amaliy masalalarni yechish jarayonlarning komputer texnologiyalari asosida o'rganadi.

Ushbu o'quv qo'llanmani yozilishida Namangan davlat universiteti "Informatika", "Matematika", "Oliy matematika" kafedralarining professor-o'qituvchilarining maslahatlaridan foydalanildi. Mualliflar kitob qo'lyozmasi bilan tanishib, foydali maslahatlar bergan professorlar R.Ibragimov, N.Otaxonov, f-m.f.d. professor Yu.Apakovga, pedagogika fanlari doktori, dotsent X. Najmiddinovaga, dotsentlar A.Mashrabboyev, M.Xolmurodov, G.Yunusova, Sh. Boltaboev, M.Dadaxonov, M. Eshnazarovalarga chuqur minnatdorchilik izhor qilamiz.

O'quv qo'llanma bo'yicha takliflar, aniqlangan kamchiliklar bo'yicha fikr mulohazalaringizni afsona77@list.ru Elektron pochtasiga yuborishingizni so'raymiz. Siz bildirgan tanqidiy fikr-mulohazalaringizni kutib qolamiz.

Mualliflar

Amaliy mashg'ulot №1

Mavzu: Amal xatoliklarini baholash

Mashg'ulotning maqsadi: Talabalarda xatoliklarni baholash ko'nikmasini shakllantirish

Mashg'ulotning jihozi:

a) Ko'rgazmali material

b) Kompyuterning dasturiy ta'minoti

c) Matematik tizimlar

Mashg'ulot texnik vositalari: Pentium IV kompyuterlari (qo'shimcha qurilmalari) bilan jihozlangan kompyuter sinfi

Mashg'ulotning borishi:

a) Auditoriya va talabalarni darsga tayyorligini aniqlash

b) Yo'qlama qilish.

c) Talabalarda amal xatoliklari bo'yicha bilimlarini aniqlash

d) Yangi mavzuning bayoni: kompyuter va ko'rgazmali qurollar yordamida bayon etiladi (Aqliy hujum metodi)

e) Dars yakuni: Amaliy mashg'ulot bo'yicha har bir talabaga individual topshiriqlar beriladi.

Mavzuning qisqacha bayoni.

A-aniq son, a-uning taqribiy qiymati bo'lsin, yani, $a \approx A$. U holda taqribiy sonning absolyut va nisbiy xatoliklari tushunchalari kiritiladi.

1) $a \approx A$, $\Delta a = |A-a|$-absl ut xato, $\delta a = \Delta a / |a|$-nisbiy xato, $\Delta a \le \Delta_a$-limit absolyut xato, $\delta a \le \delta_a$- limit nisbiy xato. $A = a \pm \Delta_a$-kelishuv.

2) Funksiya va argument xatoliklari: $u = f(x) = f(x_1,...,x_n)$, $X_i = x_i \pm \Delta_{x_i}$.

$$\Delta_u = \sum_{i=1}^{n} \left|\frac{\partial f(x)}{\partial x_i}\right| \Delta_{x_i}$$ -funksiyaning limit absolut xatoligi,

$$\Delta_{x_i} = \Delta_u / (n \left|\frac{\partial f(x)}{\partial x_i}\right|), i = 1..n$$,-argumentning limit absolut xatoligi.

3) Yaxlitlash xatosi. $a = a_m a_{m-1} .. a_0, a_{-1} a_{-2} .. a_{-n}$, biror m+n+1 xonali haqiqiy son bo'lsin. Sonning o'nli yozuvidagi har qanday 0 dan farqli raqami *muhim raqam*

deyiladi. Ikkita muhim raqamlar orasidagi nollar ham muhim deyiladi, muhim raqam keyinadagi nol ham muhim raqam deyiladi. Nolga teng bo'lmagan raqamlar oldidagi nollar muhim bo'lmaydi. Agar shu yozuvda sonning absolyut xatosi verguldan keyingi n – raqamining bir birligidan oshmasa, (n – raqamining bir birligining yarmidan oshmasa) keng ma'noda m+n+1 ta *ishonchli raqamlarga* ega deyiladi (tor ma'noda m+n+1ta ishonchli raqamlarga ega deyiladi).

Taqribiy son shunday yoziladiki, unda ishonchli raqamlar saqlanadi. *Sonni biror raqamining 1 birligigacha aniqlik bilan yaxlitlash (keng ma'noda ishonchli raqamlar saqlash) uchun* shu raqamni o'ng tamondagi barcha raqamlar o'chiriladi. Natijada vujudga kelgan son o'chirilmay qolgan raqamning 1 birligidan oshmaydi.

Sonni biror raqamining 1 birligining yarmigacha aniqlik bilan yaxlitlash (tor ma'noda ishonchli raqamlar saqlash) uchun shu raqamdan o'ngda turgan raqamlar o'chiriladi va a) o'chirilgayotgan raqamlarning birinchisi 5 dan katta bo'lsa, saqlanayotgan oxirgi raqamga 1 qo'shiladi, b) o'chirilgayotgan raqamlarning birinchisi 5 dan kichik bo'lsa o'zgartirilmaydi, v) o'chirilayotgan raqamning birinchisi 5 bo'lib, qolganlarini ichida 0 dan farqlilari bo'lsa, oxirgi raqamga 1 qo'shiladi, g) o'chirilgayotgan raqamlarning birinchisi 5 va qolganlari 0 bo'lsa saqlanayotgan son toq bo'lsa unga 1 qo'shiladi, juft bo'lsa qo'shilmaydi.

Taqribiy sonning limit absolyut xatosi bilan ishonchli raqamlari orasida munosabat mavjud: $\delta_a \leq 10^{1-n} a_m^{-1}$ (keng ma'noda). Agar taqribiy son a ikkitadan ko'p, yani $n \geq 2$ ishonchli raqamlarga ega bo'lsa, ushbu baho o'rinli: $\delta_a \leq 0.5 * 10^{1-n} a_m^{-1}$ (tor ma'noda). Aksincha, agar sonning limit absolyut nisbiy xatosi ushbu

$$\delta_a \leq 2(a_m + 1)^{-1} * 10^{1-n}$$

tengsizlikni qanoatlantirsa, a son tor ma'noda n ta ishonchli raqamga ega.

Masalalarni Mathcadda yechish namunalari.

1-masala. $A = \sqrt{30} = 5.44722557505...$, $a = 5.447, \Delta_a = ?, \delta_a = ?$.

$$\Delta a=|A-a|=|5.44722557505-5.447|=0.000225...\le 0.00023=\Delta_a.$$

$$\delta_a=\frac{\Delta_a}{|a|}=\frac{0.00023}{5.447}=0.00004222507\le 0.00004223=\delta_a=0.004223\%.$$

$$A=\sqrt{30}=5.44722\pm 0.00023.$$

2-masala. Funksiya ko'rinishi va argumentlarning qiymati va xatoliklari berilgan, funksiyaning yo'l qo'yilishi mumkin bo'lgan xatosi topilsin.

$$u(m,n,k)=m^2n^3/\sqrt{k}\quad m=28.3\pm 0.02\quad n=7.45\pm 0.01\ k=0.678\pm 0.003.$$

Topish kerak: $\delta m, \delta n. \delta k, \Delta u=?$.

$$>u(m,n,k):=m^2n^3/\sqrt{k}\quad u=4.022E5$$

$$>\delta m:=\frac{\Delta m}{|m|}\quad \delta n:=\frac{\Delta n}{|n|}\quad \delta k:=\frac{\Delta k}{|k|}$$

$$\Delta u:=\left|\frac{\partial u(m,n,k)}{\partial m}\right|\Delta m+\left|\frac{\partial u(m,n,k)}{\partial n}\right|\Delta n+\left|\frac{\partial u(m,n,k)}{\partial k}\right|\Delta k$$

$$>\delta m=7.067E-4\quad \delta n=1.342E-3\quad \delta k=4.425E-3\quad \Delta u=7.653E-3$$

3-masala. Oldingi misolda funksiya berilgan va uning xatoligi berilgan, argumentlarning mumkin bo'lgan xatoliklari topilsin. $\Delta u=0.001, \delta m, \delta n, \delta k=?$. Ushbu formulaga ko'ra topamiz:

$$\Delta m:=\frac{\Delta u}{3\left|\frac{\partial f(x)}{\partial m}\right|}\quad \Delta n:=\frac{\Delta u}{3\left|\frac{\partial f(x)}{\partial n}\right|}\quad \Delta k:=\frac{\Delta u}{3\left|\frac{\partial f(x)}{\partial k}\right|}$$

$$\Delta m=0.036\quad \Delta n=6.335E-5\quad \Delta k=3.459E-3.$$

4-masala. Agar $r=15\pm 0,02, h=19,1\pm 0,05, \pi=3,14$, bo'lsa konus hajmi qanday absolut va nisbiy xatoliklar bilan hisoblanishini aniqlang.

Yechish. Ma'lumki,

$v=\pi r^2h/3=4498,1,\ \partial v/\partial\pi=r^2h/3=1432,5,\ \partial v/\partial r=2\pi rh=599,74,\ \partial v/\partial h=\pi r^2/3=235,5$ ekanligi uchun limit absolyut va nisbiy xatoliklar formulasiga asosan:

$$\Delta_v=\left|\frac{\partial v}{\partial\pi}\right|\Delta_\pi+\left|\frac{\partial v}{\partial r}\right|\Delta_r+\left|\frac{\partial v}{\partial h}\right|\Delta_h=26,06,\delta_v=\frac{26,1}{4498}=0,006\to v=4498\pm 26,1.$$

5-masala.To'g'ri to'rtburchakning uzini 0,1 limit absolyut xatolik bilan hisoblash uchun uning tomonlarini qanday limit absolyut xatolik bilan hisoblash zarur?, $a \approx 4, b \approx 5$ m.

Yechish. Ravshanki, $S = ab, \Delta_S = 0{,}1$. Xatoliklar nazariyasining teskari masalasi formulalaridan topamiz:

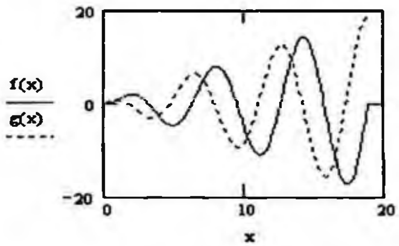
$$\left|\frac{\partial S}{\partial x_i}\right| \Delta_{x_i} = \frac{\Delta_S}{n}, \Delta_{x_i} = \frac{\Delta_S}{n\,|\partial S/\partial x_i|}, \ \frac{\partial S}{\partial a} = b = 5, \frac{\partial S}{\partial b} = a = 4, \ \Delta_a = \frac{0{,}1}{2*5} = 0{,}01, \ \Delta_b = \frac{0{,}1}{2*4} = 0{,}0125.$$

Matematik tizimlarda grafiklar yaratish.

Mathcadda $f(x) = x\sin(x), g(x) = x\cos(x)$ egri chiziqlar grafiklarini chizamiz:

$f(x) := x\sin(x) \quad g(x) := x\cos(x)$ // *functsiyalarni berish*

$a := 0 \quad b := \pi \quad n := 100 \quad h := \dfrac{b-a}{n} \quad x := a, a+h..b$ // *oraliq, nuqtalar soni, qadamlar*



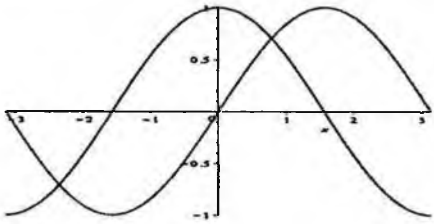
1.1-rasm.Mathcaddagi chizmasi

Insert (Alt+I)>Graph>X-Y Plot (Shift+@) menuning chiziqlar chizish qismini ochilsa shablon chiqadi. Undan so'ng quyi marker o'rniga x ni, chap yonboshdagi marker o'rniga funktsiyalar nomini kiritamiz. Bir necha funktsiya vergul bilan ajratiladi. Kursorni grafikdan tashqariga olib chiqilsa grafiklarni ekranga chiqaradi. Graph>Surface Plot (Ctrl+2) komandasi orqali sirtlarni chizadi.

Mapleda egri chiziqlar grafiklarini plot komandasi bilan chizish.

Bitta grafikda bir necha chiziqni chizish mumkin.

Misol1. plot([sin(x), cos(x), x = -3.14 .. 3.14,['red','blue]');



1.2-rasm. Mapledagi chizmasi

Qo'shimcha misollar yechish.

1) $X = m^2 n^3 / \sqrt{k}, m = 28.3 \pm 0.02, n = 7.45 \pm 0.01, k = 0.678 \pm 0.003$;

2) $N = (n-1)(m+n)/(m-n)^2, m = 5.72 \pm 0.02, n = 3.0567 \pm 0.0001$;

3) $V = \pi h^2 (R - h/3), h = 11.8 \pm 0.02, R = 23.67$.

Yechish. 1) Quyidagi ishlarni bajaramiz:

$m^2 = 800,9; n^3 = 413,5; \sqrt{k} = 0.8234; X = 800.9 * 413.5 / 0.8234 = 402200 = 4.02 * 10^5$.

$\delta_m = 0.02/28.3 = 0.00071; \delta_n = 0.01/7.45 = 0.00135; \delta_k = 0.003/0.678 = 0.00443.$

$\delta_X = 2\delta_m + 3\delta_n + 0.5\delta_k = 0.00142 + 0.00405 + 0.00222 = 0.00769 = 0.77\%$.

Shunday qilib topamiz: $\Delta_X = 4.02 * 10^5 \mp 3.1 * 10^3. \delta_X = 0.77\%$.

Mavzu bo'yicha topshriqlar

1-misol. $f(n) = A_n = \sqrt[k]{n}, k = 2,3; n = 1..N,$ funksiya berilgan. Uni 2, 4, 6, 8 xona bilan yaxlitlab funksiyaning absolyut va nisbiy xatoliklari topilsin.

Izoh 1. Agar $n = i^2 \Rightarrow k = 3$ va $n \neq i^2 \Rightarrow k = 2$ deb olinishi kerak.

Izoh 2. Agar n=1 bo'lsa $A = \pi = 3,1415926535897932384 62$ deb olinsin.

2-misol. Funksiya qiymatining, asolyut, nisbiy xatoliklar topilsin.

Ro'yxatdagi tartib raqam bo'yicha topshriqlar tanlanadi.

1.1-tablisa

№	Funksiya	1-o'zgaruvchi	2-o'zgaruvchi	3-o'zgaruvchi	4-o'zgaruvchi
1	$u=ab/\sqrt[3]{c}$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
2	$u=[(a+b)c/d^2$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
3	$u=(a^2+4ab+b^2)/(c+d)^2$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
4	$u=\sqrt{ab}/\sqrt[3]{c}$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
5	$u=a^3(b+c)/\sqrt[3]{d}$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
6	$u=(a+b)c^3/\sqrt[3]{d}$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
7	$u=\sqrt{ab}/\sqrt[3]{cd}$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
8	$u=(a+b)c/d^2$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
9	$u=(a+b)^2/(c^2+d^2)$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
10	$u=a^2b/\sqrt[3]{c}$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
11	$u=(a+b)/\sqrt[3]{c-d}$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
12	$v=hS(1+a/A+a^2/A^2)/3$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
13	$u=ab^3/\sqrt[3]{c}$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
14	$u=(a-b)\sqrt[3]{c}/\sqrt{m+n}$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
15	$v=h^2(a^2+4ab+b^2)/18(a+b)^2$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
16	$u=ab/c^2$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
17	$u=(a+b)/\sqrt{(c-d)m}$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
18	$v=\pi h/(3a^2+h^2)/6$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
19	$v=\pi^2Dd^2/4$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
20	$u=\pi\sqrt{D^4-d^4}/64$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
21	$u=a^2(1+2b/c+g^2/c^2)$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
22	$u=a^2b/c^3$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
23	$u=m\sqrt{a-b}/(c+d)$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005
24	$v=\pi h(2D^2+Dd+0.75d^2)$	3.85 ± 0.01	5.86 ± 0.02	4.82 ± 0.003	0.82 ± 0.005

25	$u=\sqrt{cd/b}$	3.85±0.01	5.86±0.02	4.82±0.003	0.82±0.005
26	$u=\sqrt[3]{a-b}/(m(m-a))$	3.85±0.01	5.86±0.02	4.82±0.003	0.82±0.005
27	$u=\sqrt{p(p-a)(p-b)(p-c)}$	3.85±0.01	5.86±0.02	4.82±0.003	0.82±0.005
28	$u=Qc^3/48E$	3.85±0.01	5.86±0.02	4.82±0.003	0.82±0.005
29	$q=(2n-1)^2(x+y)/(x-y)$	3.85±0.01	5.86±0.02	4.82±0.003	0.82±0.005
30	$v=\beta(\alpha b-\beta a)/b^2(b+\beta)$	3.85±0.01	5.86±0.02	4.82±0.003	0.82±0.005

Nazariy savollar?

1. Absolyut xato, chegaraviy absolyut xato nima, nisbiy xato, chegaraviy nisbiy xato nima?

2. Nisbiy va absolyut xatoliklar orasida qanday munosabat mavjud?

3. Funksiya va argumentlar xatoliklari orasida qanday munosbat mavjud?

4.Xatoliklar nazariyasining to'g'ri va teskari masalasiga doir elementar geometriyadan misollar keltiring.

Amaliy mashg'ulot №2

Mavzu: Chiziqli tenglamalar sistemasini iterasiya usuli bilan yechish.

Mashg'ulotning maqsadi: Talabalarda chiziqli tenglamalar sistemasini ko'nikmasini shakllantirishdan iborat.

Mashg'ulotning jihozi:

a) Ko'rgazmali material

b) Kompyuterning dasturiy ta'minoti

c) Matematik tizimlar

Mashg'ulot texnik vositalari: Pentium IV kompyuterlari (qo'shimcha qurilmalari) bilan jihozlangan kompyuter sinfi

Mashg'ulotning borishi:

a) Auditoriya va talabalarni darsga tayyorligini aniqlash

b) Yo'qlama qilish.

c) Talabalardan o'tilgan mavzularni takrorlash

d) Yangi mavzuning bayoni: kompyuter va ko'rgazmali qurollar yordamida bayon etiladi (Assesment metodi)

e) Dars yakuni: Amaliy mashg'ulot bo'yicha har bir talabaga individual topshiriqlar beriladi.

Mashg'ulotning qisqacha bayoni.

1. Chiziqli algebra masalalari: $Ax=b, B=A^{-1}, x=A^{-1}b, \Delta=\det(A), Ax=\lambda x, x, \lambda=?$.

2.CHATSni yechish usullari: Gauss, teskari matrisa usuli, Kramer qoidasi, Xoleskiy usullari.

3.Uch diagonalli sistemani yechish uchun haydash (progonka) usuli $Ax=b$:

$$b_0x_0+c_0x_1=d_0, a_ix_{i-1}+b_ix_i+c_ix_{i+1}=d_i,\ i=1,...n-1, a_nx_{n-1}+b_nx_n=d_n:$$

$$\begin{bmatrix} b_0 & c_0 & 0 & . & 0 & 0 & 0 \\ a_1 & b & c & . & 0 & 0 & 0 \\ 0 & a & b & . & 0 & 0 & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & b_{n-2} & c_{n-2} & 0 \\ 0 & 0 & 0 & . & a_{n-1} & b_{n-1} & c_{n-1} \\ 0 & 0 & 0 & . & 0 & a_n & b_n \end{bmatrix} * \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ . \\ x_{n-2} \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ . \\ d_{n-2} \\ d_{n-1} \\ d_n \end{bmatrix}$$

Yechimlar orasida quyidagi bog'lanish mavjud: $x_i=u_{i+1}x_{i+1}+v_{i+1}$, i=n-1,..,0. Haqiqatan ham, 0- va n-tenglamalarda ikkitadan nomalumlar istirok etayapti. 0-tenlamani 1-tenglamaga, 1-tenglamani 2-tenglamaga, va hokazo, i-tenglamani i+1-tenglamaga qo'yib, i=0..n-1, bunday munosabatlarni olamiz. $u_i, v_i, i=0..n$ sonlar haydash (progonka) koeffisientlari deyiladi. Ular quyidagicha formulalar bilan topiladi (to'gri yurish):

$$u_{i+1}=\frac{-c_i}{a_iu_i+b_i}, v_{i+1}=\frac{d_i-a_iv_i}{a_iu_i+b_i}, i=0,..,n-1, u_0=v_0=0;$$

So'ng, no'malumlar quyidagi formulalar bilan topiladi (teskari yurish):

$$x_n=\frac{d_n-a_nv_n}{a_nu_n+b_n},\ x_i=u_{i+1}x_{i+1}+v_{i+1},\ i=n-1,..,0$$

4.Iterasiya usuli: taqribiy yechim $x^{(k)}\approx\xi, \lim_{k\to\infty}x^{(k)}=\xi, f(x^{(k)})=Ax^{(k)}-f\to 0$:

$Ax=b \Leftrightarrow x=Cx+d, x^k=Cx^{k-1}+d. \lim_{k\to\infty}x^k=\xi, A\xi=b$.bu yerda $x=g(x)=Cx+d$-iterator, yani iterasiyalovchi funksiya.

Yakobi iterasiya usuli: $x_i^{(k)} = (b_i - \sum_{j \neq i}^{n} a_{ij} x_j^{(k-1)}) / a_{ii}$, k=1,2,....

Zeydel iterasiya usuli: $x_i^{(k)} = (b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k-1)}) / a_{ii}$ k=1,2,....

5.Yaqinlashish haqidagi asosiy teoremalar.

Teorema1. Agar $q = \|C\| < 1$ bo'lsa iterasiyalar yaqinlashadi: $x^k = Cx^{k-1} + d, \lim_{k \to \infty} x^k = \xi, A\xi = b$

Teorema2. Agar S salmoqli bosh diognalga ega bo'lsa Yakobi, Zeydel iterasiya usullari yaqinlashadi. Zeydel usuli Yakobi usulidan tezroq yaqinlashadi.

Teorema3. Agar $Ax = b$ CHATSda matrisa A simmetrik, musbat aniqlangan bo'lsa oddiy iterasiya, Zeydel iterasiya usullari yaqinlashadi. Bunday matrisali CHATS normal CHATS deyiladi.

Ixtiyoriy $A_1 x = b_1$ CHATSni normal ko'rinishga quyidagicha keltirish mumkin: $A_1^* A_1 x = A_1^* b_1 \Leftrightarrow Ax = b, A = A_1^* A_1, b = A_1^* b_1$ (Gauss almashtirishi).

Teorema4. $q = \|C\| < 1 \Leftrightarrow |\lambda| < 1, \forall \lambda : Cx = \lambda x$.

Teorema5. Iterasiyalarning qoldiq hadlari uchun ushbu baholar o'rinli:

$$\|\xi - x^{(k)}\| \le \|C\|^k \|\xi - x^{(k-1)}\|, \|\xi - x^{(k)}\| \le \frac{\|C\|}{1 - \|C\|} \|x^{(k)} - x^{(k-1)}\|, \|C\| = \max_{\|x\|=1} \|Cx\|.$$

6. Vektor va matrisalarning mos normalari.

$\|A\| = \max_{\|x\|=1} \|Ax\|$ miqdor $x \in R^n$ vektor normasiga mos matrisaning normasi deyiladi. Quyida vektorlar, matrisalarning mos normalarini ushbu jadvalda keltiramiz:

2.1-tablisa

№	Vektor normasi	Mos matrisa normasi	iterasiya usulining yaqinlashish sharti
1	$\|x\|_\infty = \max_{i=1..n} \|x_i\|$	$\|A\|_\infty = \max_{i=1..n} \sum_{j=1}^{n} \|a_{ij}\|$	$\sum_{j=1, j \neq i}^{n} \|a_{ij}\| < \|a_{ii}\|, i = 1..n$
2	$\|x\|_1 = \sum_{j=1}^{n} \|x_j\|$	$\|A\|_1 = \max_{j=1..n} \sum_{i=1}^{n} \|a_{ij}\|$	$\sum_{i=1, i \neq j}^{n} \|a_{ij}\| < \|a_{jj}\|, j = 1..n$

3	$\|x\|_2=\sqrt{\sum_{j=1}^{n}\|x_j\|^2}$	$\|A\|_2=\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\|a_{ij}\|^2}$	$\|A\|_2<1$
4	$\|x\|_3=\sqrt{\sum_{j=1}^{n}\|x_j\|^2}$	$\|A\|=\max_{i=1,n}\sqrt{\lambda_{max}},\lambda_{max}=\max \|A\|(A^T A)$	

7. A matrisa uchun tasvir: $A:=D-L-U$:

$$L=-\begin{bmatrix}0&0&0&0\\a_{21}&0&0&0\\a_{31}&a_{32}&.&0\\a_{n1}&a_{n2}&a_{nn-1}&0\end{bmatrix},U=-\begin{bmatrix}0&a_{12}&.&a_{1n}\\0&0&.&a_{2n}\\0&0&.&a_{n-1n}\\0&0&0&0\end{bmatrix},D=\begin{bmatrix}a_{11}&0&0&0\\0&a_{12}&0&0\\0&0&.&0\\0&0&0&a_{nn}\end{bmatrix}.$$

8. Sodda iterasiya (Yakobi) usuli:

$$x_i^{(k)}=\frac{1}{a_{ii}}(b_i-\sum_{j\neq i}^{n}a_{ij}x_j^{(k-1)}),\ k=0,1,....$$

$$x^{(k)}=dJ+CJx^{(k-1)},dJ=D^{-1}b,CJ=D^{-1}(L+U),k=0,1,....$$

9. Zeydel usuli:

$$x_i^{(k)}=\frac{1}{a_{ii}}(b_i-\sum_{j=1}^{i-1}a_{ij}x_j^{(k)}-\sum_{j=i+1}^{n}a_{ij}x_j^{(k-1)}),\ k=0,1,....\ .$$

$$x^{(k)}=dS+CSx^{(k-1)},dS=(D-L)^{-1}b,CS=(D-L)^{-1}U,k=0,1,...$$

10. Relaksasiya usuli.

$C\Omega:=(D-\omega L)^{-1}[(1-\omega)D+\omega U]\quad d\Omega:=\omega(D-\omega L)^{-1}b\quad z^{<k+1>}:=d\Omega+C\Omega z^{<k>}$

Topshiriqni Mathcad dasturida bajarish.

$$A:=\begin{bmatrix}40&1&1&1\\2&40&2&1\\2&2&40&2\\1&1&2&40\end{bmatrix}\qquad b:=\begin{bmatrix}43\\45\\46\\44\end{bmatrix}$$

Ushbi misolni mathcad dasturida yeching.

Mathcad dasturida CHATSni yechish uchun ushbu ichki funksiyalar mavjud: lsolve(A,b), rref(B), B=[A|b],Given..Find, $x=A^{-1}b$, Minimize, Mineer, augment, submatrix, stack, norm1, norm2, norme. Ushbu ishki buyruqlar orqali misolni matcad dasturida yechilgan.

1. Gauss usulining ichki funksiyasi rref(A) yordamida yechimni topish

$$ORIGIN := 1 \quad B := augment(A, B) \quad B = \begin{bmatrix} 40 & 1 & 1 & 1 & 43 \\ 2 & 40 & 2 & 1 & 45 \\ 2 & 2 & 40 & 2 & 46 \\ 1 & 1 & 2 & 40 & 44 \end{bmatrix} \quad // \textit{ kengaytirilgan matrisa}$$

$$s := rref(B) \quad s = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad x := submatrix(B,1,4,5,5) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad // \textit{ yechimni ajratish}$$

2. Teskari matrisa usulida yechimni topish

$$A := \begin{bmatrix} 40 & 1 & 1 & 1 \\ 2 & 40 & 2 & 1 \\ 2 & 2 & 40 & 2 \\ 1 & 1 & 2 & 40 \end{bmatrix} \quad b := \begin{bmatrix} 43 \\ 45 \\ 46 \\ 44 \end{bmatrix} \quad y := A^{-1}b \quad D := |A| \quad D = 2.539 * 10^6 \quad y^T = [1\ 1\ 1\ 1]$$

3. Ichki funksiya lsolve(A,b) yordamida yechimni topish:

$$r := lsolve(A, b) \quad r^T = [1\ 1\ 1\ 1]$$

4. Kramyer qoidasi:

$$A1 := \begin{bmatrix} 43 & 1 & 1 & 1 \\ 45 & 40 & 2 & 1 \\ 46 & 2 & 40 & 2 \\ 44 & 1 & 2 & 40 \end{bmatrix} A2 := \begin{bmatrix} 40 & 43 & 1 & 1 \\ 2 & 45 & 2 & 1 \\ 2 & 48 & 40 & 2 \\ 1 & 44 & 2 & 40 \end{bmatrix} A3 := \begin{bmatrix} 40 & 1 & 43 & 1 \\ 2 & 40 & 45 & 1 \\ 2 & 2 & 48 & 2 \\ 1 & 1 & 44 & 40 \end{bmatrix} A4 := \begin{bmatrix} 40 & 1 & 1 & 43 \\ 2 & 40 & 2 & 45 \\ 2 & 2 & 2 & 48 \\ 1 & 1 & 2 & 44 \end{bmatrix}$$

$$D := |A| \quad D1 := |A1| \quad D2 := |A2| \quad D3 := |A3| \quad D4 := |A4| \quad i := 1..4 \quad z_i := D_i / D \quad z = [1\ \ 1\ \ 1\ \ 1]$$

5. Teskari matrisa va uni tekshirib ko'rish:

$$C = \begin{bmatrix} 0.0251 & -0.0006 & -0.0006 & -0.0006 \\ -0.0012 & 0.0251 & -0.0012 & -0.0005 \\ -0.0012 & -0.0012 & 0.0251 & -0.0012 \\ -0.0005 & -0.0006 & -0.0012 & 0.0251 \end{bmatrix} C * A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A * C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6. Xoleskiy usuli. A matrisani LU usulda ko'paytuvchilarga ajratish:

$$Q := lu(A) \quad Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 40 & 1 & 1 & 1 \\ 0.05 & 1 & 0 & 0 & 0 & 39.95 & 1.95 & 0.95 \\ 0.05 & 0.049 & 1 & 0 & 0 & 0 & 39.855 & 1.904 \\ 0.025 & 0.024 & 0.048 & 1 & 0 & 0 & 0 & 39.86 \end{bmatrix}$$

Bu yerda L-bosh diogonali 1 ga teng chap quyi uchburchak, U-o'ng uqori uchburchak matrisalar. $Ax = b$ sistema quyidagicha yechiladi:

$$Ax = LUx = b \Rightarrow Ly = b, Ux = y,$$

L:=submatrix(Q,1,4,5,8) $L=\begin{bmatrix}1 & 0 & 0 & 0\\ 0.05 & 1 & 0 & 0\\ 0.05 & 0.049 & 1 & 0\\ 0.025 & 0.024 & 0.048 & 1\end{bmatrix}$ $y:=L^{-1}*b$

U:=submatrix(Q,1,4,9,12) $U=\begin{bmatrix}40 & 1 & 1 & 1\\ 0 & 39.95 & 1.95 & 0.95\\ 0 & 0 & 39.855 & 1.904\\ 0 & 0 & 0 & 39.86\end{bmatrix}$ $x:=U^{-1}*y$ $x=\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$

7. Uch diognalli CHATS uchun progonka usuli.

$$n:=3\quad A:=\begin{bmatrix}40 & 1 & 0 & 0\\ 2 & 40 & 2 & 0\\ 0 & 3 & 40 & 3\\ 0 & 0 & 4 & 40\end{bmatrix}\quad d:=\begin{bmatrix}41\\44\\46\\44\end{bmatrix}\quad a_0:=0\ c_n:=0\ i:=0..n\ b_i:=A_{i,i}\ b^T=\begin{bmatrix}40 & 40 & 40 & 40\end{bmatrix}$$

$i:=0..n-1\ c_i:=A_{i,i+1}\ c^T=\begin{bmatrix}1 & 2 & 3 & 0\end{bmatrix}\ ,i:=1..n\ a_i:=A_{i,i-1}\ a^T=\begin{bmatrix}0 & 2 & 3 & 4\end{bmatrix}$

$x:=A^{-1}d\ x=\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}$

$u_0:=0\ v_0:=0\ i:=0..n-1\ u_{i+1}:=-c_i/(a_iu_i+b_i)\ v_{i+1}:=(d_n-a_nv_n)/(a_iu_i+b_i)$

$x_n:=(d_n-a_nv_n)/(a_nu_n+b_n)\ x_n=1\ i:=1..n-1\ x_{i+1}:=u_{i+1}x_{i+1}+v_{i+1}\ x^T=\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}$

8. Determinantni hisoblash: $D:=|A|\ d:=|C|\ D*d=1\ D=2.539*10^6$

9. Yakobi (sodda) va Zeydel, relaksasiya va iterasiya usullari.

$$A:=\begin{bmatrix}40 & 1 & 1 & 1\\ 2 & 40 & 2 & 1\\ 2 & 2 & 40 & 2\\ 1 & 1 & 2 & 40\end{bmatrix}\quad b:=\begin{bmatrix}43\\45\\46\\44\end{bmatrix}\quad E:=identity(4)\quad E=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\quad x^{<1>}:=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}\quad y^{<1>}:=x^{<1>}$$

$$D:=\begin{bmatrix}40 & 0 & 0 & 0\\ 0 & 40 & 0 & 0\\ 0 & 0 & 40 & 0\\ 0 & 0 & 0 & 40\end{bmatrix}\quad L:=\begin{bmatrix}0 & 0 & 0 & 0\\ -2 & 0 & 0 & 0\\ -2 & -2 & 0 & 0\\ -1 & -1 & -2 & 0\end{bmatrix}\quad U:=\begin{bmatrix}0 & -1 & -1 & -1\\ 0 & 0 & -2 & -1\\ 0 & 0 & 0 & -2\\ 0 & 0 & 0 & 0\end{bmatrix}\quad k:=0..10$$

$TJ:=D^{-1}(L+U)\ dJ:=D^{-1}b\ x^{<k+1>}:=dJ+TJx^{<k>}$ (*Yakobi usuli*)

$TS:=(D-L)^{-1}U\ dS:=(D-L)^{-1}b\ y^{<k+1>}:=dS+TSy^{<k>}$ (*Zeydel usuli*)

$T\Omega:=(D-\omega L)^{-1}[(1-\omega)D+\omega U]\ d\Omega:=\omega(D-\omega L)^{-1}b\ z^{<k+1>}:=d\Omega+T\Omega z^{<k>}$ (*relaksasiya usuli*)

Izoh. Relaksasiya usuli quyidagicha yaratiladi:

$$Ax=b\Rightarrow(D-L-U)x=b\Rightarrow\omega(D-L-U)x=\omega b\Rightarrow-\omega Lu=\omega(U-D)x+\omega b\Rightarrow$$
$$Dx-\omega Lux=Dx+\omega(U-D)x+\omega b\Rightarrow(D-\omega L)x=[(1-\omega)D+\omega U]x+\omega b\Rightarrow$$
$$x=T\Omega x+d\Omega, T\Omega=(D-\omega L)^{-1}[(1-\omega)D+\omega U], d\Omega=\omega(D-\omega L)^{-1}b\Rightarrow$$
$$x^{(k)}=d\Omega+T\Omega x^{(k-1)}, k=1,2...$$

Bu iterasiyalarning yaqinlashish sharti: A matrisa salmoqli diognalga ega.

2.2-tablisa. Yakobi usuli

		1	2	3	4	5	6	7	8	9	10
x =	1	0	1.075	0.991	1.001	1	1	1	1	1	1
	2	0	1.125	0.986	1.002	1	1	1	1	1	1
	3	0	1.15	0.984	1.002	1	1	1	1	1	1
	4	0	1.1	0.988	1.001	1	1	1	1	1	...

2.3-tablisa. Zeydel usuli

		1	2	3	4	5	6	7	8	9	10
y =	1	0	1.075	0.997	1	1	1	1	1	1	1
	2	0	1.071	0.997	1	1	1	1	1	1	1
	3	0	1.07	1.001	1	1	1	1	1	1	1
	4	0	0.993	1	1	1	1	1	1	1	...

2.4-tablisa Relaksasiya usuli

		1	2	3	4	5	6	7	8	9	10
z =	1	0	0.86	0.981	0.998	1	1	1	1	1	1
	2	0	0.866	0.983	0.998	1	1	1	1	1	1
	3	0	0.868	0.986	1	1	1	1	1	1	1
	4	0	0.811	0.963	0.993	0.999	1	1	1	1	...

10.CHATSni normal holga keltirib iterasiya usulini qo'llash. $Ax=b$ sistemada A matrisa salmoqli diognalga ega bo'lmasa, uni normal holga keltiriladi: $A^T Ax = A^T b$ (Gauss almashtirishi) va yangi CHATS ga Yakobi yoki Zeydel iterasiya usuli qo'llaniladi. Normal CHATS da A matrisa quyidagi 2 ta xossaga ega:

1) A matrisa simmetrik: $(Ax, y) = (x, Ay), \forall x, y \in R^n$;

2) A matrisa musbat aniqlangan: $A^T A \geq 0 \Leftrightarrow (A^T Ax, x) = (Ax, Ax) \geq 0, \forall x \in R^n$.

Ixtiyoriy $Ax=b$ CHATS ni normal ko'rinishga quyidagicha keltiriladi: $A^T Ax = A^T b$, va yangi CHATS ga iterasiya usullari qo'llaniladi.

Misol:

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} \; b := \begin{bmatrix} 6 \\ 7 \\ 6 \end{bmatrix} \; A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 4 & 1 \end{bmatrix} \; C := A^T A \;\; d := A^T b \;\; C = \begin{bmatrix} 14 & 10 & 14 \\ 10 & 9 & 12 \\ 14 & 12 & 26 \end{bmatrix} \; d = \begin{bmatrix} 38 \\ 31 \\ 52 \end{bmatrix}$$

$$D:=\begin{bmatrix}14&0&0\\0&9&0\\0&0&26\end{bmatrix}\quad L:=\begin{bmatrix}0&0&0\\-10&0&0\\-14&-12&0\end{bmatrix}\quad U:=\begin{bmatrix}0&-10&-14\\0&0&-12\\0&0&0\end{bmatrix}$$

$$CC:=(D-L)^{-1}U \quad dC:=(D-L)^{-1}d \quad t:=\begin{bmatrix}0\\0\\0\end{bmatrix} \quad k:=1..30 \quad t^{<k+1>}:=dC+CC\,t^{<k>} \quad t=$$

2.5-tablisa

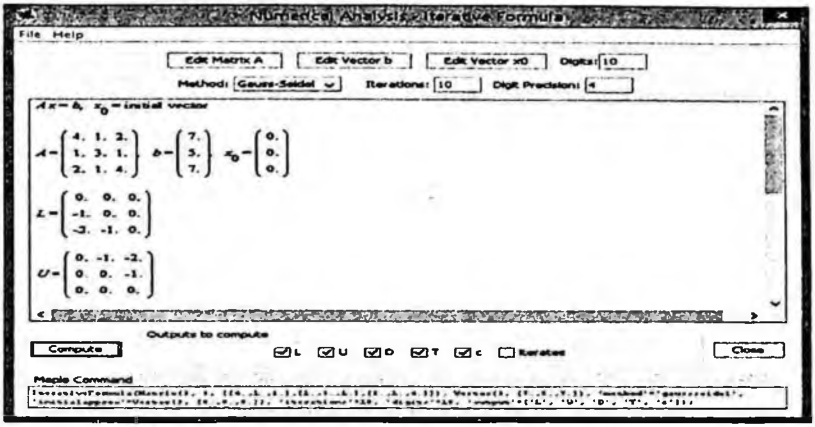
$t=$	22	23	24	25	26	27	28	29	30	31
1	0.997	0.997	0.998	0.998	0.999	0.999	0.999	0.999	1	1
2	1.003	1.002	1.002	1.001	1.001	1.001	1.001	1.001	1	1
3	1	1	1	1	1	1	1	1	1	...

Topshiriqni Maple dasturida yechish.

Yuqoridagi barcha iterasiya usulari Maple tizimida ham bajarilishi mumkin. Maple tizimida masalani 2 xil usulda yechish mumkin: komandalar yordamida va interaktiv usul yordamida. Biz interaktiv usulda yechishni ko'ramiz. Maple tizimining interaktiv imkoniyatlari Tools>Numerical Analysis>Iterative Formula komandasini menyu yordamida berish bilan bajariladi. Natijada muloqot oynasi ochiladi va unda zarur komandalarni sichqoncha bilan tanlash mumkin. Bu yerda matrisaning o'lchami, uning o'zini, Chiziqli sistemaning o'ng tomon vektorini, boshlang'ich iterasiya vektorini, iterasiyalar sonini, iterasiya usullaridan Yakobi, Zeydel, relaksasiyani tanlash mumkin. Oynada ayrim parametrlarni chiqarishni berkitish mumkin. Eng quyida Maple komandalarini ko'rsatish maydonchasi mavjud. Barcha parametrlar tanlangach Computer komandasi bilan barcha tanlovlar asosida hisoblashni ishga tushirish mumkin. Natijada biz quyidagi yechimni olamiz:

$$A:=\begin{bmatrix}4&1&2\\1&3&1\\2&1&4\end{bmatrix} b=\begin{bmatrix}7\\5\\7\end{bmatrix} x_0=\begin{bmatrix}0\\0\\0\end{bmatrix} L=\begin{bmatrix}0&0&0\\-1&0&0\\-2&-1&0\end{bmatrix} U=\begin{bmatrix}0&-1&-2\\0&0&-1\\0&0&0\end{bmatrix} D=\begin{bmatrix}4&0&0\\0&3&0\\0&0&4\end{bmatrix}$$

$$x_{(k+1)}=Tx_{(k)}+c \quad T=(D-L)^{-1}U=\begin{bmatrix}0&-.2500&-.5000\\0&.8333e-1&-.1667\\0&.1042&.2917\end{bmatrix} C=(D-L)^{-1}b=\begin{bmatrix}1.750\\1.083\\.6042\end{bmatrix} x_{(10)}=\begin{bmatrix}1.000\\1.000\\1.000\end{bmatrix}$$



2.1-rasm

Maple dasturida CHATSni yechish uchun universal komandalar solve, fsolve, minimize ichki funksiyalari mavjud.

Misol 1.

with(linalg);

A:=matrix(2, 2, [[1, 2], [2, 1]]); B := inverse(A); $A := \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ $B := \begin{bmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{bmatrix}$

b := vector(2, [3, 3]); // $b := [3 \ \ 3]$ x := linsolve(A, b); // $x := [1 \ \ 1]$

y := multiply(B, b); // $y := [1 \ \ 1]$

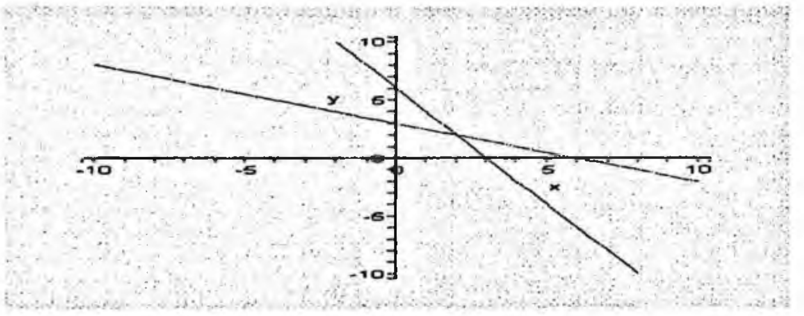
eqs := {t1+2*t2-3, 2*t1+t2-3}; // $eqs := \{t1 + 2 * t2 - 3, 2 * t1 + t2 - 3\}$

z := solve(eqs, [t1, t2]); // $z := [[t1 = 1, \ t2 = 1]]$

Misol. 2. Chiziqli tenglamalar sistemasini yechish.

s1:={2*x+y=6, x+2*y=6}: solve(s1,{x, y}); //{y=2,x=2}

with(plots):implicitplot(s1, x=-10..10, y=-10..10);



2.2-rasm

Mavzu bo'yicha topshiqlar.

1.Berilgan CHATS Yakobi, Zeydel, relaksasiya iterasiya usullari bilan echilsin. Yechim Mathcad, Maple dasturlarida ichki funksiyalarda va algoritm tuzib olinsin. Natijalarning mosligiga yerishilsin.

2. Ixtiyoriy matrisali CHATS normal ko'rinishga keltirilib Zeydel iterasiya usuli bilan echilsin bu erda n-ni o'rniga ro'yxatdagi tartib raqam qo'yiladi.

$$A=\begin{bmatrix} n+3 & 1 & 1 & 1 \\ 1 & n+3 & 2 & 1 \\ 1 & 1 & n+3 & 3 \\ 1 & 1 & 4 & n+3 \end{bmatrix},\ n=1,2,3,...,\ b=\begin{bmatrix} n+6 \\ n+7 \\ n+8 \\ n+9 \end{bmatrix}.$$

Nazariy savollar va topshiriqlar

1.CHATS nima. Aniq yechim nima.Yechim qachon mavjud, mavjudmas va yagona.

2.CHATS qanday aniq usullar bilan echiladi va eng asosiy usul qaysi usul?

3.CHATS ni taqribiy yechish qanday g'oyaga asoslangan?

4. Iterator nima?

5. Iterasiyalarning yaqinlashish sharti nima?

6.CHATS uchun sodda iterasiya usuli. Iterasiyalarning yaqinlashishining yetarli sharti?

7.Zeydel, relaksasiya iterasiya usullari va iterasiyalarning yaqinlashishi.

8.Normal CHATS uchun iterasiya usuli, uning yaqinlashishi. CHATSni normallashtirish?

Xos sonlarni topish.

1.Matrisaning xos sonlari va vektorlari: $\lambda, x: Ax = \lambda x, x \neq 0$. Matrisaning asriy (xarakteristik) tenglamasi:

$$Ax = \lambda x, x \neq 0 \Rightarrow Ax - \lambda x = 0, x \neq 0 \Leftrightarrow D(\lambda) = |A - \lambda E| = (-1)^n(\lambda^n - p_1\lambda - .. - p_n) = 0.$$

Asriy tenglama ildizlari-bu xos sonlardir.

2. Simmetrik matrisa xos sonlari va vektorlari xossalari:

$$1)\lambda \in R,\ 2)Ax = \lambda x, Ay = \mu y, \lambda \neq \mu \rightarrow (x, y) = 0.$$

Xos sonlarni to'liq topish usullari

1) Ekstremal xos sonlar uchun iterasiya usuli:

$$x^{(k)}=A^k x^{(1)}, y^{(k)} = (A^{-1})^k y^{(1)}, k = 2,3,..., \lim_{k\to\infty} y_i^{(k)} / y_i^{(k-1)} = \lambda_{min}, \lim_{k\to\infty} x_i^{(k)} / x_i^{(k-1)} = \lambda_{max}, i = 1,...,n\ .$$

2) Xos sonlarni determinantni bevosita hisoblab topish usuli.

Bu ish 2 etapda olib boriladi:

1-etap. Asriy tenglama koeffisientlari topilib u quriladi:

$$D(\lambda) = |A - \lambda E| = (-1)^n(\lambda^n - p_1\lambda^{n-1} - .. - p_n) = 0$$

2-etap. Asriy tenglama echilib xos sonlar aniqlanadi:

$$D(\lambda_i) = |A - \lambda_i E| = 0, i = 1..m \leq n$$

3) Asriy tenglama qurishning Leverre usuli

1) matrisaning darajalari hisoblansin: $A^k, k = 1..n$;

2) ularning izlari hisoblansin: $A^k : s_k = s(A^k) = a_{11}^{(k)} + .. + a_{nn}^{(k)}$;

3) Nyuton formulalari asosida koeffisientlar hisoblansin:

$p_1 = s_1, p_k = (s_k - p_1 s_{k-1} - .. - p_{k-1}s_1)/k, k = 1..n$;

4) xos sonlar asriy tenglamadan topilsin: $D(\lambda)=(-1)^n(\lambda^n - p_1\lambda^{n-1} - ... - p_{n-1}\lambda - p_n) = 0$.

4) Asriy tenglama $D(\lambda) = 0$ qurishning interpolyasiya usuli.

1) sonlar olamiz $\lambda_i = i, i = 1..n$.

2) determinantlarni hisoblaymiz: $d_i = D(i)$, $i=1, ..., n$;

3) CHATS quramiz: $D(i) = |A - i * E| = (-1)^n * (i^n - \sum_{j=1}^{n} p_j i^{n-j}) = d_i, i = 1..n$:

$$\begin{bmatrix} p_1+...+p_n=1^n-(-1)^n d_1, \\ p_1 2^{n-1}+p_2 2^{n-2}+...+p_n=2^n-(-1)^n d_2, \\ .... \\ p_1 n^{n-1}+p_2 n^{n-2}...+p_n=n^n-(-1)^n d_n \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & 1 & . & 1 \\ 2^{n-1} & 2^{n-2} & . & 1 \\ . & . & . & . \\ n^{n-1} & n^{n-2} & . & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ . \\ p_n \end{bmatrix} = \begin{bmatrix} 1^n-(-1)^n d_1 \\ 2^n-(-1)^n d_2 \\ . \\ n^n-(-1)^n d_n \end{bmatrix}.$$

Belgilashlar kiritamiz va bu CHATSni quyidagicha yozib olamiz: $Mp=b$:

$$M=[m_{ij}]=[i^{n-j}],i,j=1,...,n,p=[p_1,p_2,...,p_n],b=[b_1,b_2,...,b_n],b_i=i^n-(-1)^n d_i$$

4) Bu sistemani yechib asriy tenglama koeffisientlarni topamiz: $p=M^{-1}b$.

5) Asriy tenglamani yechib xos sonlarni topamiz.

5) Asriy tenglama qurishning Krilov usuli.

Faraz qilaylik: $D(\lambda)=|A-\lambda E|=(-1)^n(p_1+p_2\lambda+...+p_{n-1}\lambda^{n-2}+p_n\lambda^{n-1}+\lambda^n)=0$.

Gamilton-Kelli teoremasiga asosan, matrisa o'z asriy tenglamasini qanoatlantiradi:

$$D(A)=0: D(A)=|A-AE|=|0|=0.$$

Shuning uchun egamiz:

$$D(A)=|A-AE|=(-1)^n(p_1+p_2A+...+p_{n-1}A^{n-1}+p_nA^n)=0 \Rightarrow (p_1+p_2A+...+p_{n-1}A^{n-2}+p_nA^{n-1}+A^n)=0$$

Bu yerdan $p_1E+p_2A+...+p_nA^{n-1}=-A^n$ deb olamiz. Bu tenglamani ixtiyoriy nolmas vektorga $x\in R^n$ ko'paytirib vektor tenglama hosil qilamiz:

$$p_1Ex+p_2Ax+...+p_nA^{n-1}x=-A^nx.$$

Vektorlar kiritib $M^{<k>}:=A^{k-1}*x, k=1..n, d=-A^nx$, asriy tenglama koeffisientlari uchun ushbu chiziqli tenglamalar sistemasini olamiz:

$$\begin{bmatrix} M_{11} & M_{12} & . & M_{1n} \\ M_{21} & M_{22} & . & M_{2n} \\ . & . & . & . \\ M_{n1} & M_{n2} & . & M_{nn} \end{bmatrix} * \begin{bmatrix} p_1 \\ p_2 \\ . \\ p_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ . \\ d_n \end{bmatrix}, M_{ij}=(A^{j-1}*x)_i, d=-A^nx=-\begin{bmatrix} (A^nx)_1 \\ . \\ (A^nx)_n \end{bmatrix}$$

Uni yechib asriy tenglama koeffisientlarini topamiz. Asriy tenglamani yechib xos sonlarni topamiz.

6) LU-algoritm (Rutisxauzer, 1958).

Bu eng ishonchli, lekin uncha turg'un bo'lmagan algoritm. Usulning algoritmining g'oyasi quyidagidan iborat: 1) A matrisa $A=A_1:=LU$ shaklga keltiriladi, L-quyi chap uchburchak, U- diogonali 1 ga teng o'ng uqori uchburchak matrisa; 2) A matrisaning xos sonlarini o'zgartirmaydigan o'xshash almashtirishlar bajariladi: $A_{m+1} := UL = UA_m U^{-1}, m = 1, 2, ... 3$. Bu ketma-ketlik limiti simmetrik matrisalar uchun bosh diogonalda A matrisaning xos sonlarini beradi. Ixtiyoriy matrisa uchun bu limit ikki o'lchovli blokli matrisaga intiladi. Bu bloklarning xos sonlari A matrisaning kompleks xos sonlariga yaqinlashadi.

7) QR-algoritm (Frensis, 1961).

Bu eng ishonchli, eng turg'un algoritm. Usul algoritmining g'oyasi quyidagidan iborat: 1) A matrisa $A = A_1 := QR$ shaklga keltiriladi, Q-ortogonal matrisa, R-o'ng uqori uchburchak matrisa; 2) A matrisaning xos sonlarini o'zgartirmaydigan o'xshash almashtirishlar bajariladi: $A_{m+1} := Q^{-1} A_m Q, m = 1,2,...3$) Bu ketma-ketlik limiti simmetrik matrisalar uchun bosh diogonalda A matrisaning xos sonlarini beradi. Ixtiyoriy matrisa uchun bu limit ikki o'lchovli blokli matrisaga intiladi. Bu bloklarning xos sonlari A matrisaning kompleks xos sonlariga yaqinlashadi.

MATHCADDA YIRIKLASHTIRILGAN ALGORITMLAR

1) Xos sonlarni *eigenvals(A)* **va xos vektorlarni** *eigenvecs(A)* **ichki funksiyalar bilan topish:**

$$A := \begin{bmatrix} 5 & 1 & 4 \\ 3 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix} \; r := eigenvals(A) \; s := eigenvecs(A) \; s = \begin{bmatrix} -0.326 & -0.702 & -0.557 \\ -0.548 & -0.094 & -0.371 \\ -0.77 & 0.603 & 0.743 \end{bmatrix}, r = \begin{bmatrix} 21.405125 \\ 13.594875 \\ 13 \end{bmatrix}$$

2) Xos sonlarni topishning determinantni bevosita hisoblash usuli.

Bu bizning asosiy usul:

$$ORIGIN := 1 \; A := \begin{bmatrix} 5 & 1 & 4 \\ 3 & 3 & 3 \\ 1 & 2 & 1 \end{bmatrix} \; E := identity(3) \; D(\lambda) := |E - \lambda * E| \; D(\lambda) \rightarrow -\lambda^3 + 9 * \lambda^2 - 12 * \lambda + 6$$

$$p := D(x) coeff, x \rightarrow \begin{bmatrix} 6 \\ -12 \\ 9 \\ -1 \end{bmatrix} \; s := polyroots(p) \; s = \begin{bmatrix} 0.746 + 0.493i \\ 0.746 - 0.493i \\ 0.7508 \end{bmatrix}$$

3) Leverre usulining algoritmi.

ORIGIN:=1 n:=3 A:= $\begin{bmatrix}5&1&4\\3&3&2\\1&2&1\end{bmatrix}$ k=1..n s_i:=tr(A^i) s=[9 57 423] p_1:=s_1 *//matrisa, izlar*

k:=2..n $p_k := \frac{1}{k}(s_k - \sum_{i=1}^{k-1} p_i s_{k-i})$ $p = \begin{bmatrix}9\\-12\\6\end{bmatrix}$ $q := -\begin{bmatrix}p_3\\p_2\\p_1\end{bmatrix}$ $q = \begin{bmatrix}-6\\12\\-9\end{bmatrix}$ *// $D(\lambda)$ koeffisientlari*

$P := stack(q,1)$ $P^T = [-6\ 12\ -9\ 1]$ *// $D(\lambda) = |A - \lambda E|$ ning barcha koeffisientlari*

r:=polyroots(P) $r = \begin{bmatrix}0.746+0.493i\\0.746-0.493i\\7.508\end{bmatrix}$ r1:=eigenvals(A) $r1 = \begin{bmatrix}0.746+0.493i\\0.746-0.493i\\7.508\end{bmatrix}$ //javob

4) Xos sonlar uchun interpolyasiya usuli. Usulning g'oyasi: $D(i) = |A - i*E| = (-1)^n*(i^n - \sum_{j=1}^{n} p_j i^{n-j}) = d_i, i = 1..n$. Bu bizga algoritm beradi:

ORIGIN:=1 n:=3 E:=identity(n) A:= $\begin{bmatrix}5&1&4\\3&3&2\\1&2&1\end{bmatrix}$ i:=1..n j:=1..n $D_i := |A - i*E|$ $d_i := (-1)^n * D_i$

$d = \begin{bmatrix}-2\\-10\\-24\end{bmatrix}$ $M_{i,j} := i^{n-j}$ $M = \begin{bmatrix}1&1&1\\4&2&1\\9&3&1\end{bmatrix}$ $b_i := i^n - d_i$ $b = \begin{bmatrix}3\\18\\51\end{bmatrix}$ *//CHATS:matrisa,o'ng tomon*

$p := M^{-1} * b$ $p^T = [9\ \ -12\ \ 6]$ $p1^T = -[p_3\ \ p_2\ \ p_1]$ *// $D(\lambda) = |A - \lambda E|$ koeffisientlari*

$q := stack(p1,1)$ $q^T = [-6\ 12\ -9\ 1]$ *// asriy tenglama $D(\lambda) = |A - \lambda E|$ koeffisientlari*

$r := polyroots(q)$ $r = \begin{bmatrix}0.746+0.493i\\0.746-0.493i\\7.508\end{bmatrix}$ $r1 := eigenvals(A)$ $r1 = \begin{bmatrix}7.508\\0.746+0.493i\\0.746-0.493i\end{bmatrix}$ // javoblar

5) Xos sonlari topishning Krilov usuli. Yiriklashtirilgan algoritm.

ORIGIN:=1 n:=3 $A := \begin{bmatrix}5&1&4\\3&3&2\\1&2&1\end{bmatrix}$ $x := \begin{bmatrix}1\\0\\0\end{bmatrix}$ $E := identity(n)$ k:=1..n

$M^{\langle k\rangle} := A^{\langle k-1\rangle} * x$ $M = \begin{bmatrix}1&5&32\\0&3&26\\0&1&12\end{bmatrix}$ $d := (-1)*A^n*x$ $p := M^{-1}*d$ $d^T = -[234\ \ 198\ \ 96]$

$p^T = [-6\ \ 12\ \ -9]$ $P := stack(p,1)$ $P^T = [-6\ 12\ 9\ 1]$

r:=polyroots(P) s:=eigenvals(A) $r=\begin{bmatrix}0.746+0.493i\\0.746+0.493i\\7.508\end{bmatrix}$ $s=\begin{bmatrix}7.508\\0.746+0.493i\\0.746-0.493i\end{bmatrix}$

6) QR-algoritmning yiriklashtirgan ko'rinishi (Mathcad uni bajaradi).

ORIGIN:=1 *//QR-algoritm*

$A:=\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}$ $p:=eigenvals(A)$ $p=\begin{bmatrix}16.117 & -1.117 & 0\end{bmatrix}^T$ $Q:=identity(3)$ $R:=identity(3)$

$M:=qr(A)$ $r:=rows(M)$ $s:=cols(M)$ $r=3$ $s=6$ $n:=10$ *// qr – алмаштириш*

```
QR(A,n,Q,R,M) := | for k ∈ 1..n
                 |   | M ← qr(A)
                 |   | Q ← submatrix(M,1,r,1,r)
                 |   | R ← submatrix(M,1,r,r+1,2·r)
                 |   | A ← Q^-1·A·Q
                 | Q
                 | R
                 | M
```

(4)

$Q=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$ $R=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$ $z:=QR(A,n,Q,R,M)$ $z=\begin{pmatrix}1&0&0&16.11684&-4.89898&0\\-0&1&0&0&-1.11684&0\\0&0&1&0&0&0\end{pmatrix}$

Bu yerda xos sonlar o'ng tomondagi R matrisaning diogonalida joylashgan.

7) Ekstremal xos sonlar uchun iterasiya usuli. Xos sonlarning qismiy problemasi. Mathcadda ichki funksiyalar va algoritm asosida xos sonlarni topish:

ORIGIN:=1 $A:=\begin{bmatrix}34&1&2\\1&35&2\\1&3&36\end{bmatrix}$ $C:=A^{-1}$ $x^{\langle 1\rangle}:=\begin{bmatrix}1\\0\\1\end{bmatrix}$ $y^{\langle 1\rangle}:=\begin{bmatrix}1\\0\\1\end{bmatrix}$ i:=1..3 k=1..30

$s:=eigenvals(A)$ $r:=eigenvals(C)$ $s=\begin{bmatrix}38.44949\\33.55051\\33\end{bmatrix}$ $r=\begin{bmatrix}0.026008\\0.029806\\0.030303\end{bmatrix}$

$x^{\langle k\rangle}:=A^k*x^{\langle 1\rangle}$ $y^{\langle k\rangle}=C*x^{\langle 1\rangle}$ $\lambda_i:=\frac{x_{i,30}}{x_{i,29}}$ $\mu_i:=\frac{y_{i,30}}{y_{i,29}}$ $\lambda=\begin{bmatrix}38.261594\\38.499143\\38.448885\end{bmatrix}$ $\mu=\begin{bmatrix}0.029779\\0.030411\\0.030553\end{bmatrix}$ $\lambda_{max}=38.449$ $\lambda_{min}=33$

Mavzu bo'yicha topshiriqlar

Berilgan matrisa (belgilash Mapleniki) $A = matrix([n+3,1,1;1,n+4,2;1,1,n+5]), n = 1,2,...$ Matrisaning xos sonlarini bevosita tarif bo'yicha, Mathcad komandalari asosida, Leverre, interpolyasiya, Krilov, LU, QR usullar yordamida toping. Natijalarning adekvatligiga yerishilsin. Bu yerda n-ning o'rniga talabaning ro'yxatdagi tartib raqamini n-ning o'rniga qo'yib ishlanadi.

Nazariy savollar va topshiriqlar

1.Matrisaning xos soni nima?

2.Simmetrik matrisaning xos sonlarining xossalari nima?

3.Xarakteristik tenglama nima?

4.Xos vektor nima?

5.Xos sonlar bevosita tarif asosida qanday ketma-ketlikda topiladi?

6.Leverre usuli nima?

7.Krilov usuli nima?

8. Interpolyasiya usuli nima?

9.LU-algoritm nima?

10.QR-algoritm nima?

Amaliy mashg'ulot №3

Mavzu: Bir nomalumli algebraik tenglamalarni taqribiy yechish

Mashg'ulotning maqsadi: Talabalarda bir nomalumli algebraik tenglamalarni taqribiy yechish ko'nikmasini shakllantirish

Mashg'ulotning jihozi:

a) Ko'rgazmali material

b) Kompyuterning dasturiy ta'minoti

c) Matematik tizimlar

Mashg'ulot texnik vositalari: Pentium IV kompyuterlari (qo'shimcha qurilmalari) bilan jihozlangan kompyuter sinfi

Mashg'ulotning borishi:

a) Auditoriya va talabalarni darsga tayyorligini aniqlash

b) Yo'qlama qilish.

c) Talabalardan o'tilgan mavzularni takrorlash

d) Yangi mavzuning bayoni: kompyuter va ko'rgazmali qurollar yordamida bayon etiladi (Keys stadi metodi)

e) Dars yakuni: Amaliy mashg'ulot bo'yicha har bir talabaga individual topshiriqlar beriladi.

Mavzuni qisqacha bayoni

1.Nochiziq tenglama to'g'risida umumiy tushunchalar: f(x)=0, $f(\xi)=0$.

Tenglamani f(x)=b, $f(\xi)=b$ ko'rinishlarda ham yozish mumkin.

2.Ildizlarni ajratish masalasi $f(a)f(b)<0 \triangleright \exists c: f(c)=0$.

Kesmani 2 ga bo'lish usuli: $x^{(k)}=x^{(k-1)}+sign(f(a)f(x^{(k-1)}))(b-a)/2^k$, $k=1,2,...,x^{(0)}=a$.

$$|\xi-x^{(k)}|\le(b-a)/2^k.$$

3.Taqribiy yechimlar $x^{(k)}$ iterasiya usuli bilan topiladi: $x^k=g(x^{k-1})\approx\xi, \xi=\lim\limits_{k\to\infty}x^k, f(\xi)=0$. Bunda $\varepsilon_k=|\xi-x^{(k)}|$–taqribiy yechim xatoligi deyiladi. $r_k=f(x^{(k)})$-miqdor esa taqribiy yechimning tenglamani qanoatlantirganlik darajasini ko'rsatib, chetlanish deyiladi.

4. Iterasiya usuli bilan f(x)=0 tenglamani taqribiy yechish:

$$f(x)=0 \Leftrightarrow x=g(x),\ g(x)=x-\lambda f(x),\ q=\max_{x\in[a,b]}|g'(x)|<1,\ sign(\lambda f'(x))>0,$$

$$\lambda=\pm 2/(m+M),\ M=\max_{x\in[a,b]}f'(x),\ m=\min_{x\in[a,b]}f'(x),$$

$$x^k=g(x^{k-1})\approx\xi, \xi=\lim_{k\to\infty}x^k, f(\xi)=0. |\xi-x^{(k)}|\le q^k|\xi-x^{(0)}|, |\xi-x^{(k)}|\le\frac{q}{1-q}|x^{(k)}-x^{(k-1)}|.$$

$f(x)=0 \Leftrightarrow x=g(x),\ g(x)=x-\lambda f(x),\ q=\max\limits_{x\in[a,b]}|g'(x)|<1$ almashtirishda tenglamaninig x=g(x) ko'rinishi uni iterasiya usulini qo'llash uchun qulay ko'rinish deyiladi.

Demak, iterasiyalar taqribiy yechim ekan: $\xi=\lim\limits_{k\to\infty}x^k, f(\xi)=0$. Funksiya $g(x)=x-\lambda f(x)$ iterasiyalovchi funksiya-iterator deyiladi.

5. Nyuton iterasiya usuli. Soddalashgan Nyuton usuli.

$x_{k+1}=g(x_k)=x_k-f(x_k)/f'(x_k)$,- Nyuton iterasiya usuli,

$y_{k+1} = g(y_k) = y_k - f(y_k)/f'(y_0), x_k \approx \xi, y_k \approx \xi, k = 0,1,...$ -soddalashgan Nyuton usuli,

$$\left|\xi - x^{(k)}\right| \le (p\left|\xi - x^{(0)}\right|)^{2^k} / p, p = 2M_2 / m_1, m_1 = \min\left|f'(x)\right|, M_2 = \max\left|f''(x)\right|.$$

6. Mathcad ning ichki funksiyalari: bir nomalumli nochiziq tenglamalarni yechish uchun standart ichki funksiyalar polyroots(v) (ko'phadli tenglama $f(x)=a_0+a_1x+...+a_nx^n$, $v=[a_0,a_1,...,a_n]^T$ uchun), root(f(x),x)) (ixtiyoriy tenglama uchun). Yana oddiy iterasiya va Nyuton iterasiya usullarini tashkil etish mumkin.

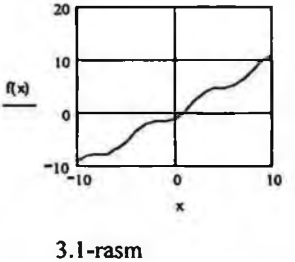
Topshiriqlarni Mathcadda bajarish.

Misol 1. root(f(x),x)) ichki funksiyasi. f(x)=x-cos(x)=0 tenglamani yechish quyidagicha amalga oshiriladi:

x:=0 f(x):=x-cos(x) *// Boshlang'ich iterasiya va tenglama*

$r:=root(f(x),x)^T$ r=0.7398 *// Ichki funksiyaga murojaat va ildizni chiqarish*

Grafikni keltiramiz (f(x)=x-cos(x), $-10 \le x \le 10$).Tenglama yagona yechimga ega:



3.1-rasm

Misol 2. Ko'phadli tenglama uchun poliroots(v) ichki funksiyasi.

$f(x) := a(x-k)^3 + b(x-k)^2 + c(x-k) = 0, a,b,c \in R, k = n \in N$ tenglamaning barcha ildizlarini topilsin.Tenglamani 3 ta haqiqiy yechimlari bor:

$a := 1\ b := 3\ c := 1\ n := 4\ k := n\ \ f(x) := a(x-k)^3 + b(x-k)^2 + c(x-k)$

$p(x) := f(x)\text{exp}and \to x^3 - 9x^2 + 25x - 20\ \ v := p(x)coeffs \to [-20\ 25\ -9\ 1]^T$

$r := polyroots(v)\ r = [1.382\ 3.618\ 4]^T$

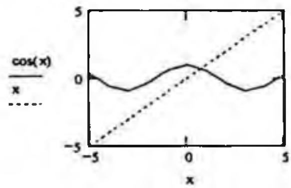
$p(x) := x^3 - 9x^2 + 25x - 20\ \ s := root(p(x), x) \to [4\ 1.3819\ 3.6150]$

Misol 3. Iterasiya usulini qurish. $f(x)=x-\cos(x)=0$ tenglamani qaraymiz va iterasiya usuli uchun yiriklashgan algoritm tuzamiz:

$x_0:=0 \quad k:=0..10$ *// Boshlang'ich iterasiya va iterasiyalar sonini berish*

$x_{k+1}:=\cos(x_k)$ *// f(x)=0 ni x=g(x) ko'rinishga keltirib iterasiyalar qurish*

x^T=(1, 0.5403, 0.8576,..., 0.7314) *// Natijani chiqarish*



3.2-rasm

Misol 4. Yana o'sha tenglamani qaraymiz. Faqat, p.3 dagi iteratorni olamiz.

$M:=2\ m:=1\ l:=2/(M+m)\ l=2/3\ f(x):=x-\cos(x)\ g(x):=x-l*f(x)\ x_0:=0\ k:=0..10\ x_{k+1}:=g(x_k)$

3.1-jadval

$x^T=$

	0	1	2	3	4	5	6	7	8	9	10
0	0	0.667	0.746	0.738	0.739	0.739	0.739	0.739	0.739	0.739	0.739

iterasiyalar oldingi variantga nisbatan tezroq yaqinlashyapti.

Misol 5. Iterasiya usuli. $f(x)=x^3-5x+1=0$ tenglamani qaraymiz. Avvalo, tenglama 3 ta yechimga ega: $r:=polyroots(1\ -5\ 0\ 1)\ r=[-2.330059, 0.20164, 2.128419]^T$.

1) $\xi_1=-2.330059$ yechim uchun iterasiyalar quramiz:

$x=g_1(x)=\sqrt[3]{5x-1})/5, q_1=\max|g_1'(x)|=\max\left|(5/3)\sqrt[3]{(5x-1)^2}\right|<(5/3\sqrt[3]{81})<1, x\in[-2,-3]$

$x_0:=0\ \ k:=0..4$ *// Boshlang'ich iterasiya va iterasiyalar soni*

$x_{k+1}:=\sqrt[3]{5x_k-1}$ *// f(x)=0 ni x=g(x) ko'rinishga keltirish va iterasiyalar qurish*

Natijalarni 3.2-jadvalda ko'rish mumkin.

3.2-jadval

$x^T=$

	0	1	2	3	4	5
0	-2.5	-2.3811	-2.34562	-2.33483	-2.33152	...

2) $\xi_2 = 0.20164$ yechim uchun iterasiyalar quramiz:

$x = g_2(x) = (1+x^3)/5, q_2 = \max|g_2'(x)| = \max|3x^2/5| < 0.6, x \in [0,1]$

$x_0 := 0 \;\; k := 0..10$ *// Boshlang'ich iterasiya va iterasiyalar sonini berish*

$x_{k+1} := (1+x_k^3)/5$ // f(x)=0 *ni* x=g(x) *ko'rinishga keltirib iterasiyalar qurish*

$x^T = [0\ 0.2\ 0{,}2016\ 0.201639\ 0{,}20164]$. // *iterasiyalar*

3) $\xi_3 = 2.128419$ yechim uchun iterasiyalar quramiz

$x = g_3(x) = \sqrt[3]{5x-1})/5, q_3 = \max|g_3'(x)| = \max\left|(5/3)\sqrt[3]{(5x-1)^2}\right| < (5/3\sqrt[3]{81}) < 1, x \in [2,3]$

$x_0 := 0 \;\; k := 0..4$ *// Boshlang'ich iterasiya va iterasiyalar soni*

$x_{k+1} := \sqrt[3]{5x_k - 1}$ // f(x)=0 *ni* x=g(x) *ko'rinishga keltirib iterasiyalar qurish*

3.3-jadval

$x^T =$

	0	1	2	3	4	5	6	7	8	9	10
0	2.5	2.2572	2.1748	2.1453	2.1346	2.1307	2.1293	2.1287	2.1285	2.1285	...

Misol 6. Nyuton usuli. $f(x) = x - \cos(x) = 0$ tenglamani qaraymiz.

$x_0 := 0 \;\; k := 0..10$ *// Boshlang'ich iterasiya va iterasiyalar sonini berish*

$x_{k+1} := x_k - (x_k - \cos(x_k))/(1+\sin(x_k))$ *// Nyuton usuli* $x^{(k+1)} = x^{(k)} - f(x^{(k)})/f'(x^{(k)})$

$x^T = (1, 0.75036, 0.73911, ..., 0.73909)$ $\xi \approx 0.73909$ *// Natijani chiqarish*

3. Topshiriqlarni Maple matematik tizimida bajarish.

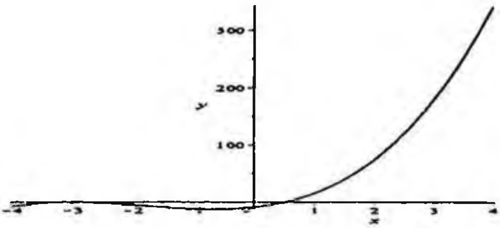
Misol 1.

p:=2*x^3+11*x^2+12*x-9:roots(p); *//[[0.5],[-3,2]](-3 2 karrali ildiz)*

solve(p(x),x);*//{x=1/2},{x=-3},{ x=-3} (-3 2 karrali ildiz)*

r:=solve(p(x),x); *// r:= {x=1/2},{x=-3},{ x=-3}*

plot(p,x=-4..4,labels=[x,y],labelfont=[TIMES,ITALIC,12]);



3.3-rasm

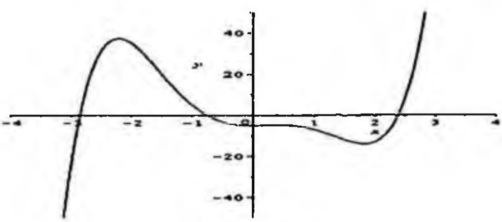
Misol 2. Ko'phadli tenglamani yechish. $f(x)=x^5-7x^3+4x^2-5=0$.

Tenglamalarni taqribiy yechish uchun universal solve(eq,x):komandaga o'xshash fsolve(eq,x) komanda ishlatiladi:

eq(x) := x^5-7*x^3+4*x^2-5; $//x^5-7x^3+4x^2-5=0$

fsolve(eq(x),{x}); //{x=-2.8608..},{x=-0.7521..},{x=2.3857..}

plot(eq(x),x=-4..4,y=-50..50,colour=[green,red]);



3.4-rasm

Misol 3. Bir necha misollarni ko'ramiz:

x:=fsolve(cos(x)=x,x); //x:=0.7390851332 (10-ta raqam bilan).

r:=solve(4*x+0.8*exp(x)-7.4561=0,x); // x:=1.200000971

x:=fsolve(4*x+0.8*exp(x)-7.4561=0,x); // x:=1.200000971

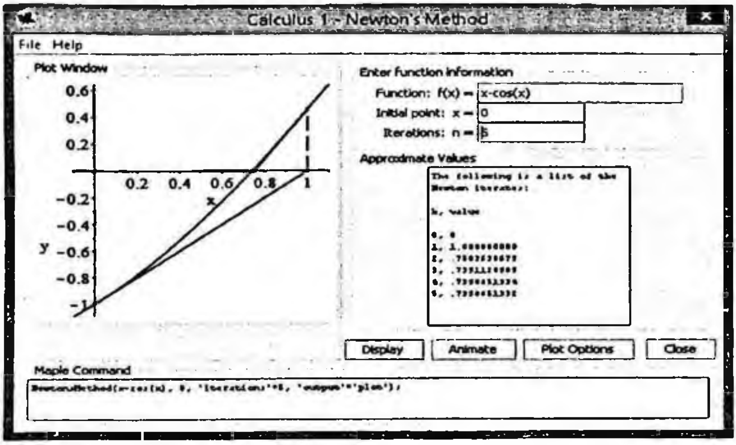
y:=fsolve(y^3-2.8*exp(y)+2.5713=0,y); // y:=-0.08545049502

q:=solve(y^3-2.8*exp(y)+2.5713=0,{y}): // q:=-0.08545049502

Misol 4. Mapleda tenglamalarni yechish uchun Nyutonning interaktiv usuli.

a) Ushbu komandani beramiz :Tools▶ Calculus▶ Single Variables ▶Newton's Methods... Interaktiv oyna paydo bo'ladi:

Bu yerda tenglama $f(x)=0$, parametr x_0, iterasiyalar soni $k:=1..n$ beriladi.



3.5-rasm

Misol 4. Tenglamalarni iterasiya va Nyuton usuli bilan yechish.

with(Student[NumericalAnalysis]);

f := x-cos(x);Newton(f, x = 1.0, tol yerance = 10^(-8)); *// 0.7390851332*

Newton(f, x = 1.0, tolerance = 10^(-8), output = sequence);

1.0, 0.7503638679, 0.7391128909, 0.7390851334, 0.7390851332 *// Nyuton usuli*

FixedPointlt yeration(f, x = 1.0, tolerance = 10^(-8), output = sequence);

1.0, 0.5403023059, 0.8575532158, 0.6542897905, 0.7934803587, 0.7013687737, 0.7639596829, 0.7221024250, 0.7504177618, 0.7314040424, 0.7442373549 *// qo'zg'almas nuqta uchun iterasiya usuli*

4. Individual topshiriqlar. Tenglamalar avvalo Mathcad va Maple dasturilarida ichki funksiyalar yordamida yechimlari topilsin, so'ng yechim algoritm tuzib iterasiya, Nyuton usullari bilan hisoblansin. Natijalarning yaqinligiga yerishilsin. Grafiklar chizilsin.

3.4-jadval

№	Tenglama	yechimlar
1	$f(x)=\sqrt{x}-x^{-1}\ln(x)+4x-1.5=0$	
2	$f(x)=\cos(x)-\exp(-x)+0.5=0$	

3	$f(x)=1.5-0.4\sqrt{x^3}-0.5\ln(x)=0$	
4	$f(x)=2-\sqrt{x^3}-2\ln(x)=0$	
5	$f(x)=1-0.5x^2\ln(x)+0.3\sqrt{x}=0$	
6	$f(x)=1-x\ln(x)+0.3\sqrt{x}=0$	
7	$f(x)=3-0.5\sqrt{x}-\exp(-0.5x^2)=0$	
8	$f(x)=3-\sqrt{x^3}+0.5\ln(x)=0$	
9	$f(x)=0.3\exp(-0.7\sqrt{x})-2x^2+4=0$	
10	$f(x)=0.5\exp(-\sqrt{x})-0.2\sqrt{x^3}+2=0$	
11	$f(x)=\exp(-0.7x)-0.3\sqrt{x}+1=0$	
12	$f(x)=3-\sqrt{x}-0.5\ln(x)=0$	
13	$f(x)=0.2\exp(-x^2)-\sqrt{x}+3=0$	
14	$f(x)=0.3\cos^2(x)-\ln(x)+2=0$	
15	$f(x)=\exp(-0,5x^2)-x^3+0.2=0$	
16	$f(x)=\exp(-0.5x)-0.2x^2+1=0$	
17	$f(x)=\exp(-0.4x^2)-0.5x^2+1=0$	
18	$f(x)=1.5-0.4\sqrt{x^3}-\exp(-x^2)\sin(x)=0$	
19	$f(x)=2-0.5x^2-0.5x^{-1}\sin(x)-x=0$	
20	$f(x)=0.3\exp(x)-\cos^2(x)+2=0$	
21	$f(x)=0.5\exp(-x^2)-x\cos(x)=0$	
22	$f(x)=\cos^2(x)-0.8x^2=0$	
23	$f(x)=1+\exp(-\sqrt{x})-\ln(x)=0$	
24	$f(x)=x\ln(x)-\exp(-0.5x^2)=0$	
25	$f(x)=\sin(0.5x)+1-x^2=0$	
26	$f(x)=\cos(0.5x)-0.4\ln(x)=0$	
27	$f(x)=\exp(-0.3x^2)-\sqrt{x}+1=0$	

28	$f(x)=\cos^2(x)-0.1\exp(x)=0$	
29	$f(x)=x^2-\exp(-x^2)=0$	
30	$f(x)=x-\sin(x)-0.25=0$	

Nazariy savollar va topshiriqlar.

1. Taqribiy yechishning g'oyasi nimadan iborat?
2. Ildizlarni ajratish nima degani va ildizlar necha xil usulda ajratiladi?
3. Ildizlarni geometrik usulda ajratish qanday g'oyaga asoslangan?
4. Iterasiya usuli nima?
5. Iterasiya usulining yaqinlashish sharti nima?
6. Tenglamani iterasiya usulini qo'llash uchun qulay ko'rinishga keltirish deganda nimani tushunasiz.

8. Nyutonning iterasiya usuli, sodda Nyuton iterasiya usuli nima.

Amaliy mashg'ulot №4

Mavzu: Chiziqsiz tenglamalar sistemasini yechish uchun oddiy iterasiya usuli

Mashg'ulotning maqsadi: Talabalarda chiziqsiz tenglamalar sistemasini yechish uchun oddiy iterasiya usuli bilan yechish ko'nikmasini shakllantirish

Mashg'ulotning jihozi:

a) Ko'rgazmali material

b) Kompyuterning dasturiy ta'minoti

c) Matematik tizimlar

Mashg'ulot texnik vositalari: Pentium IV kompyuterlari (qo'shimcha qurilmalari) bilan jihozlangan kompyuter sinfi

Mashg'ulotning borishi:

a) Auditoriya va talabalarni darsga tayyorligini aniqlash

b) Yo'qlama qilish.

c) Talabalardan o'tilgan mavzularni takrorlash

d) Yangi mavzuning bayoni: kompyuter va ko'rgazmali qurollar yordamida bayon etiladi (Aqliy hujum metodi)

e) Dars yakuni: Amaliy mashg'ulot bo'yicha har bir talabaga individual topshiriqlar beriladi.

Mavzuni qisqacha bayoni

$$f(x)=\begin{cases} f_1(x_1,...,x_n) \\ ... \\ f_n(x_1,...,x_n) \end{cases}=\begin{cases} 0 \\ . \\ 0 \end{cases}=0, x=\begin{bmatrix} x_1 \\ . \\ x_n \end{bmatrix}$$

1.Iterasiya usuli. $f(x)=0 \Leftrightarrow x=g(x), g(x)=x-Lf(x), \det(L)\neq 0, f:R^n\to R^n$ va iterasiya usuli ($g(x)=x-Lf(x)$- iterasiyalovchi funksiya, iterator) quyidagicha quriladi:

$$x^k=g(x^{k-1})\approx\xi, k=1,..., \lim_{k\to\infty}x^k=\xi, x, \xi\in R^n, \|g(x)-g(y)\|\leq q\|x-y\|, 0<q<1.$$

Bu yerda $q=\max_{x\in D}\|g'(x)\|=\max_{x\in D}\|\partial g_i(x)/\partial x_j\|$.

Iterasiyalar iterasiya usulining yaqinlashih sarti $q<1$ bajarilsagina yaqinlashadi.

2.* Qoldiq xad uchun baholar*:

$\|\xi-x^{(k)}\|\leq q^k\|\xi-x^{(0)}\|, \|\xi-x^{(k)}\|\leq q\|x^{(k)}-x^{(k-1)}\|/(1-q)=q^k\|x^{(1)}-x^{(0)}\|/(1-q)$

Mathcad dasturida chiziqsiz tenglamalar sistemasini taqribiy yechish uchun standart ichki funksiyalar mavjud, ular given ..find bloki, minimize (f(x),x), mineer(x,y) ichki funksiyasi. Undan tashqari ularni oddiy iterasiya va Nyuton iterasiya usullari yordamida ham taqribiy yechish mumkin.

Maple dasturida chiziqsiz tenglamalar sistemasini yechish uchun universal solve, fsolve, minimize ichki funksiyalari mavjud. Oddiy iterasiya va Nyuton usulini tashkil etish imkoniyati ham mavjud.

Topshiriqni Mathcadda bajarish.

Asosiy fikrlarni ushbu tenglamalar sistemasi orqali ochib beramiz:

1) $f_1(x_1,x_2)=x_1^2+x_2^2=1$, $f_2(x_1,x_2)=x_1^2-x_2=0$ (aylana va parabola, 2 ta yechim)

2) $f_1(x_1,x_2):=x_1^2-10x_1+x_2^2+8, f_2(x_1,x_2):=x_1x_2^2+x_1-10x_2+8.$

1) given ..find bloki yordamida NTS ni yechish.

x:=1 y:=0 // boshlang'ich iterasiya (x:=-1 y:=0)

Given $x^2+y^2=1$ $x^2-y=0$ *// tenglamalarni berish, barobar yo'g'on*

$r:=Find(x,y)$ *// ildizni o'zgaruvchiga berish*

$r^T=[0.7861 \quad 0.6181]$ *// ildizni chiqarish,* ($r^T=[-0.7861 \quad 0.6181]$)

iterasiya usulini tashkil etuvchi jarayonlar tuzamiz.

2) iterasiya usuli yordamida NTS ni yechish.

$x_1+0.5\cos x_2=1, \sin(x_1+1)-x_2=1.2$ sistemani qaraymiz.

ORIGIN:=1 $g(x):=[1-0.5*\cos(x_2) \quad \sin(x_1+1)-1.2]^T$ *// iterasiyalovchi funksiya*

$x^{<1>}:=[0.4 \quad 0.5]^T$ *// boshlang'ich iterasiya* $x^{<1>}:=[-0.6 \quad -0.1]^T$

$k:=1..5$ $x^{<k+1>}:=g(x^{<k>})$ *// iterasiyalarni hisoblash*

$$x=\begin{bmatrix}0.4 & 0.562 & 0.515 & 0.510 & 0.511 & 0.511\\ 0.5 & -0.215 & -0.20 & -0.202 & -0.219 & -0.219\end{bmatrix} // x=\begin{bmatrix}-0.6 & 0.502 & 0.655 & 0.510 & 0.510 & 0.510\\ -0.1 & -0.811 & -0.202 & -0.203 & -0.202 & -0.202\end{bmatrix}$$

3) Minimizasiya usuli bilan NTSni taqribiy yechish.

$f(x):=(x_1^2+x_2^2-1)^2+(x_2-x_1^2)^2\to\min$ masalani qaraymiz. Ravshanki, Minimize f(x)=0, yani f(x)=0. Shuning uchun, topamiz:

$x_1:=-1$ $x_2:=0$ *// boshlang'ich qiymat* ($x_1:=1$ $x_2:=0$)

$f(x):=[x_1^2+x_2^2-1]^2+[x_1^2-x_2]^2$ *// maqsad funksiya*

$s:=Minimize(f,x_1,x_2)$ $s^T=(-0.786\ 0.618)$ *// natijalar* $r^T=(0.787\ 0.618)$

4) Mineer ichki funksiyasi yordamida NTS ni taqribiy yechish.

$x_1:=-1$ $x_2:=0$ ($x_1:=1$ $x_2:=0$) *// Boshlang'ich qiymat*

Given $x_1^2+x_2^2-1=0, x_1^2-x_2=0$ *// Blok boshi*

$s:=Minerr(x_1,x_2)$ *// Ichki funksiyaga murojaat*

$r^T=(0.787\ 0.618)$ ($s^T=(-0.786\ 0.618)$) *// Natijalar*

Topshiriqlarni Maple dasturida bajarish.

1) NTS ni ichki funksiya bilan yechish

$f1(x1,x2):=x1^2-10x1+x2^2+8; f2(x1,x2):=x1x2^2+x1-10x2+8;$
$fsolve(\{f1,f2\},\{x1,x2\}); //\{x1=1.000000000, x2=1.000000000\}$

2)NTS ni iterasiya usuli bilan taqribiy yechish.

$f1(x1,x2):=x1^2-10x1+x2^2+8; f2(x1,x2):=x1x2^2+x1-10x2+8;$
$g1:=x1=(x1^2+x2^2+8)/10; g2:=x2=(x1x2^2+x1+8)/10;$
$fsolve(\{g1,g2\},\{x1,x2\}); //\{x1=1.000000000, x2=1.000000000\}$

3) Eng kichik kvadratlar usuli bilan taqribiy yechish

$with(Optimization): Minimize((x1^2-10x1+x2^2+8)\char`^2+(x1x2^2+x1-10x2+8)\char`^2);$
//8.33122e-19,[x=0.999986,y=0.999946].

Ko'rinib turibdiki, uch xil usul deyarli bir xil natija beryapti.

Individual topshiriqlar.

NTSning yechimi avvalo Mathcad va Maple dasturilarida ichki funksiyalar yordamida topilsin, so'ng yechim algoritm tuzib iterasiya usuli bilan hisoblansin. Natijalarning yaqinligiga yerishilsin. Grafiklar chizilsin.

4.1-tablisa

№	Tenglamalar sistemasi	Yechim
1	$\sin(x+1)-y=1.2, 2x+\cos y=2$	
2	$\sin x+2y=2, \cos(y-1)+x=0.7$	
3	$tg(xy+0.4)=x^2, 0.6x^2+2y^2=1$	
4	$\cos x+y=1.5, 2x-\sin(y-0.5)=1$	
5	$\{\sin(x+0,5)-y=1,2x-\sin(y-0,5)=1$	
6	$\cos(x+0.5)+y=0.8; \sin y-2x=1.6$	
7	$\{\cos(y-0.5)+x=0.8; \sin x+2y=1.6;$	
8	$2x-\cos(y+1)=0; y+\sin x=-0,4$	
9	$\cos(y+0,5)-x=2; \sin x-2y=1$	
10	$\sin(x+1)-y=1,2x+\cos y=2$	
11	$\sin(x+1)-y=1; 2x+\cos y=2$	
12	$\cos(x-1)+y=0,8; x-\cos y=2$	
13	$\sin x+2y=1,6; \cos(y-1)+x=1$	
14	$cox+y=1,2; 2x-\sin(y-0,5)=2$	
15	$\sin(x+y)=1,2x-0,2; x^2+y^2=1$	
16	$\cos(x-1)+y=0.5, x-\cos y=3$	

17	$\cos x + y = 1.5, 2x - \sin(y - 0,5) = 1$	
18	$\sin(x + y) - 1,6x = 0, x^2 + y^2 = 1$	
19	$\sin(x + y) - 1,2x = 0.2, x^2 + y^2 = 1$	
20	$tg(xy + 0.3) = x^2, 0.9x^2 + 2y^2 = 1$	
21	$\{\sin(x + y) - 1,3x = 0, x^2 + y^2 = 1$	
22	$tg(xy + 0,1) = x^2; 0,5x^2 + 2y^2 = 1$	
23	$\sin(x + y) = 1,1x - 0,1; x^2 + y^2 = 1$	
24	$tg(x - y) - xy = 0; x^2 + 2y^2 = 1$	
25	$\sin(x - y) - xy = -1, x^2 - y^2 = 0.75$	
26	$tg(xy + 0,2) = x^2; x^2 + 2y^2 = 1;$	
27	$\sin(x + y) - 1,5x = 0; x^2 + y^2 = 1$	
28	$\{tgxy = x^2, 0,5x^2 + 2y^2 = 1$	
29	$\sin(x + y) = 1,2x - 0,2; x^2 + y^2 = 1$	
30	$\sin(x + y) = 1,2x - 0,2; x^2 + y^2 = 1$	

Nazariy saollar va topshiriqlar.

1.Chiziqsiz tenglamalar sistemasining aniq va taqribiy yechimi nima?

2.Chiziqsiz tenglamalar sistemasi nima uchun taqribiy yechiladi?

3.Iterasiya usuli nima?

4.Chiziqsiz tenglamalar sistemasi iterasiya usulini qo'llash uchun qulay ko'rinishga keltirish deganda nimani tushunasiz?

5. Chiziqsiz tenglamalar sistemasi iterasiya usuli bilan yechganda taqribiy yechim xatoligi nima?

Amaliy mashg'ulot №5

Mavzu: Chiziqsiz tenglamalar sistemasini yechish uchun Nyuton iterasiya usuli

Mashg'ulotning maqsadi: Talabalarda chiziqsiz tenglamalar sistemasini yechish uchun Nyuton iterasiya usuli bilan yechish ko'nikmasini shakllantirish

Mashg'ulotning jihozi:

a) Ko'rgazmali material

b) Kompyuterning dasturiy ta'minoti

c) Matematik tizimlar

Mashg'ulot texnik vositalari: Pentium IV kompyuterlari (qo'shimcha qurilmalari) bilan jihozlangan kompyuter sinfi

Mashg'ulotning borishi:

a) Auditoriya va talabalarni darsga tayyorligini aniqlash

b) Yo'qlama qilish.

c) Talabalardan o'tilgan mavzularni takrorlash

d) Yangi mavzuning bayoni: kompyuter va ko'rgazmali qurollar yordamida bayon etiladi (Aqliy hujum metodi)

e) Dars yakuni: Amaliy mashg'ulot bo'yicha har bir talabaga individual topshiriqlar beriladi.

Mavzuni qisqacha bayoni

$$f(x)=\begin{cases} f_1(x_1,\dots,x_n) \\ \dots \\ f_n(x_1,\dots,x_n)\end{cases}=\begin{cases}0 \\ . \\ 0\end{cases}=0,\ x=\begin{bmatrix}x_1 \\ . \\ x_n\end{bmatrix},\ f'(x)=\begin{bmatrix} f_{1x_1}(x) & . & f_{1x_n}(x) \\ . & . & . \\ f_{nx_1}(x) & . & f_{nx_n}(x)\end{bmatrix}.$$

1. Nyuton iterasiya usullari. Bu yerda $J(x)=f'(x)=\left[\partial f_i(x)/\partial x_j\right]$-Yakobi matrisasi va Nyutonning asosiy va soddalashgan usullari quyidagicha quriladi:

$$x^{(k+1)}=g(x^{(k)})=x^{(k)}-[f'(x^{(k)})]^{(-1)}f(x^{(k)}),k=0,1,..\text{-asosiy usul,}$$

$$y^{<k+1>}:=g(y^{<k>})=y^{<k>}-(f'(y^{<0>}))^{-1}f(y^{<k>}),k=0,1,...\text{-soddalashgan usul.}$$

Usulning qoldig'i $|\xi\text{-}x^{(k)}|\le p(|\xi\text{-}x^{(k-1)}|)^2,|\xi\text{-}x^{(k)}|\le(p|\xi\text{-}x^{(0)}|)^{2^k}/p,p=M_2/(2m_1)$.

iterasiya usuli bilan aloqasi: $x^k=g(x^{k-1}),g(x)=x-[f'(x)]^{-1}f(x)$.

Mathcad dasturida chiziqsiz tenglamalar sistemasini taqribiy yechish uchun standart ichki funksiyalar mavjud, ular given ..find bloki, minimize (f(x),x), mineer(x,y) ichki funksiyasi. Undan tashqari ularni oddiy iterasiya va Nyuton iterasiya usullari yordamida ham taqribiy yechish mumkin.

Maple dasturida chiziqsiz tenglamalar sistemasini yechish uchun universal solve, fsolve, minimize ichki funksiyalari mavjud. Nyuton iterasiya usulini tashkil etish imkoniyatlari ham mavjud.

Topshiriqlarni Mathcadda bajarish.

Asosiy fikrlarni ushbu tenglamalar sistemasi orqali ochib beramiz:

1) $f_1(x_1,x_2)=x_1^2+x_2^2=1$, $f_2(x_1,x_2)=x_1^2-x_2=0$

2) $f_1(x_1,x_2):=x_1^2-10x_1+x_2^2+8, f_2(x_1,x_2):=x_1x_2^2+x_1-10x_2+8.$

1) given ..find bloki yordamida NTS ni yechish.

x:=1 y:=0 // *boshlang'ich iterasiya* (x:=-1 y:=0)

Given $x^2+y^2=1$ $x^2-y=0$ // *tenglamalarni berish, barobar yo'g'on*

$r:=Find(x,y)$ // *ildizni o'zgaruvchiga berish*

$r^T=[0.7861 \quad 0.6181]$ // *ildizni chiqarish*, ($r^T=[-0.7861 \quad 0.6181]$)

Nyuton usullarini tashkil etuvchi jarayonlar tuzamiz.

2) Nyuton iterasiya usullari yordamida NTSni taqribiy yechish.

ORIGIN:=1 $g(x)^T:=[x_1^2+x_2^2=1, x_1^2-x=0]$ // *iterasiyalovchi funksiya*

$J(x):=\begin{bmatrix} 2x_1 & 2x_2 \\ 2x_1 & -1 \end{bmatrix}$ // *Yakobi matrisasini qurish*

$x^{<1>}:=[-1 \quad 0]^T$ $y^{<1>}:=[-1 \quad 0]^T$ // *boshlang'ich iterasiyani berish* ($x^{<1>}:=[1 \quad 0]^T$)

k=1..5 $x^{<k+1>}:=x^{<k>}-(J(x^{<k>}))^{-1}f(x^{<k>})$ // *Nyuton iterasiyalarini qurish*

$x=\begin{bmatrix} -1 & -1 & -0.833 & -0.788 & -0.786 & -0.786 \\ 0 & 1 & 0.667 & 0.619 & 0.618 & 0.618 \end{bmatrix}$ // $x=\begin{bmatrix} 1 & 1 & 0.8333 & 0.7881 & 0.7861 & 0.7861 \\ 0 & 1 & 0.6667 & 0.6190 & 0.6180 & 0.6180 \end{bmatrix}$

$y^{<k+1>}:=y^{<k>}-(J(y^{<1>}))^{-1}f(y^{<k>})$ // *Soddalashgan Nyuton iterasiyalarini qurish*

$y=\begin{bmatrix} -1 & -1 & -0.833 & -0.788 & -0.786 & -0.786 \\ 0 & 1 & 0.667 & 0.619 & 0.618 & 0.618 \end{bmatrix}$ // $y=\begin{bmatrix} 1 & 1 & 0.8333 & 0.7881 & 0.7861 & 0.7861 \\ 0 & 1 & 0.6667 & 0.6190 & 0.6180 & 0.6180 \end{bmatrix}$

$x^{<k+1>}:=x^{<k>}-(J(x^{<k>}))^{-1}f(x^{<k>})$ formula hozircha faqat Mathcad da ishlayapti.

Topshiriqlarni Maple dasturida bajarish.

1) CHTS ni ichki funksiya bilan yechish

$f1(x1,x2):=x1^2-10x1+x2^2+8; f2(x1,x2):=x1x2^2+x1-10x2+8;$

$fsolve(\{f1,f2\},\{x1,x2\}); //\{x1=1.000000000, x2=1.000000000\}$

2) CHTS ni Nyuton iterasiya usuli bilan taqribiy yechish.

$f1(x1,x2):=x1^2-10x1+x2^2+8; f11(x1,x2):=2x1-10; f12(x1,x2):=2x2;$

$f2(x1,x2):=x1x2^2+x1-10x2+8; f21(x1,x2):=x2^2+1; f22(x1,x2):=2x1x2-10;$

$d(x1,x2):=f11(x1,x2)*f22(x1,x2)-f12(x1,x2)*f21(x1,x2);$

$g1:=x1=x1-d1(x1,x2)/d(x1,x2); g2:=x2=x2-d2(x1,x2)/d(x1,x2);$

$fsolve(\{g1,g2\},\{x1,x2\}); \quad //\{x1=1.000000000,\ x2=1.000000000\}$

3) Eng kichik kvadratlar usuli bilan taqribiy yechish

$with(Optimization): Minimize((x1^2-10x1+x2^2+8)\hat{}2+(x1x2^2+x1-10x2+8)\hat{}2);$

//8.33122e-19,[x=0.999986,y=0.999946].

Ko'rinib turibdiki, uch xil usul deyarli bir xil natija beryapti.

Individual topshiriqlar.

CHTSning yechimi avvalo Mathcad va Maple dasturilarida ichki funksiyalar yordamida topilsin, so'ng yechim algoritm tuzilib Nyuton usullari bilan hisoblansin. Natijalarning yaqinligiga yerishilsin. Grafiklar chizilsin.

5.1-jadval

№	Tenglamalar sistemasi	Yechim
1	$\sin(x+1)-y=1.2, 2x+\cos y=2$	
2	$\sin x+2y=2, \cos(y-1)+x=0.7$	
3	$tg(xy+0.4)=x^2, 0.6x^2+2y^2=1$	
4	$\cos x+y=1.5, 2x-\sin(y-0.5)=1$	
5	$\{\sin(x+0,5)-y=1, 2x-\sin(y-0,5)=1$	
6	$\cos(x+0.5)+y=0.8; \sin y-2x=1.6$	
7	$\{\cos(y-0.5)+x=0.8; \sin x+2y=1.6;$	
8	$2x-\cos(y+1)=0; y+\sin x=-0,4$	
9	$\cos(y+0,5)-x=2; \sin x-2y=1$	
10	$\sin(x+1)-y=1; 2x+\cos y=2$	

11	$\sin(x+1)-y=1; 2x+\cos y=2$	
12	$\cos(x-1)+y=0,8; x-\cos y=2$	
13	$\sin x+2y=1,6; \cos(y-1)+x=1$	
14	$cox+y=1,2; 2x-\sin(y-0,5)=2$	
15	$\sin(x+y)=1,2x-0,2; x^2+y^2=1$	
16	$\cos(x-1)+y=0.5, x-\cos y=3$	
17	$\cos x+y=1.5, 2x-\sin(y-0,5)=1$	
18	$\sin(x+y)-1,6x=0, x^2+y^2=1$	
19	$\sin(x+y)-1,2x=0.2, x^2+y^2=1$	
20	$tg(xy+0.3)=x^2, 0.9x^2+2y^2=1$	
21	$\{\sin(x+y)-1,3x=0, x^2+y^2=1$	
22	$tg(xy+0,1)=x^2; 0,5x^2+2y^2=1$	
23	$\sin(x+y)=1,1x-0,1; x^2+y^2=1$	
24	$tg(x-y)-xy=0; x^2+2y^2=1$	
25	$\sin(x-y)-xy=-1, x^2-y^2=0.75$	
26	$tg(xy+0,2)=x^2; x^2+2y^2=1;$	
27	$\sin(x+y)-1,5x=0; x^2+y^2=1$	
28	$\{tgxy=x^2, 0,5x^2+2y^2=1$	
29	$\sin(x+y)=1,2x-0,2; x^2+y^2=1$	
30	$\sin(x+y)=1,2x-0,2; x^2+y^2=1$	

Nazariy saollar va topshiriqlar.

1.Ildizlar analitik usulda ajratish qanday teoremaga asoslangan?

2. Ildizlarni geometrik usulda ajratish qanday g'oyaga asoslangan?

3.Chiziqsiz tenglamalar sistemasini taqribiy yechishda Nyutonning iterasiya usuli, sodda Nyuton iterasiya usuli nima.

4. Nyuton usulida taqribiy yechim xatoligi qanday?

5. Yakobi matrisasi nima va u nochiziq tenglamalar sistemasi taqribiy echilganda qaerda ishlatiladi?

Amaliy mashg'ulot №6

Mavzu: Lagranj interpolasion formulasi va uning xatoligi

Mashg'ulotning maqsadi: Talabalarda Lagranj interpolasion formulasi va uning xatoligini yechish ko'nikmasini shakllantirish

Mashg'ulotning jihozi:

a) Ko'rgazmali material

b) Kompyuterning dasturiy ta'minoti

c) Matematik tizimlar

Mashg'ulot texnik vositalari: Pentium IV kompyuterlari (qo'shimcha qurilmalari) bilan jihozlangan kompyuter sinfi

Mashg'ulotning borishi:

a) Auditoriya va talabalarni darsga tayyorligini aniqlash

b) Yo'qlama qilish.

c) Talabalardan o'tilgan mavzularni takrorlash

d) Yangi mavzuning bayoni: kompyuter va ko'rgazmali qurollar yordamida bayon etiladi (Aqliy hujum metodi)

e) Dars yakuni: Amaliy mashg'ulot bo'yicha har bir talabaga individual topshiriqlar beriladi.

Mavzuni qisqacha bayoni

1.Ko'pxad bilan interpolyasiyalash:

$$P_n(x_i) = y_i,\ \ i = 0, 1, ..., n\ ,\ P_n(x) = a_0 + a_1 x + ... + a_n x^n = ?$$

2. Lagranj interpolyasiya ko'phadi, bazis interpolyasiya ko'phad, qoldiq had:

$$L_n(x) = P_n(x) = \sum_{i=0}^{n} f(x_i) l_i(x),\ \ l_i(x) = \prod_{j=0, j\neq i}^{n} \frac{x - x_j}{x_i - x_j}, i = 0, ..., n,$$

$$R_n(f, x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x - x_0)..(x - x_n).$$

Tekis joylashgan nuqtalar $\{x_i = a + ih, i = 0..n, h = (b-a)/n\}$ uchun ega bo'lamiz:

$$L_n(x)=\omega_n(t)\sum_{i=0}^{n}\frac{f(x_i)}{(t-i)i!(n-i)!(-1)^{n-i}},\omega_n(t)=t(t-1)...(x-n),t=(x-x_0)/h,$$

$$R_n(x)=h^{n+1}\frac{t(t-1)...(t-n)}{(n+1)!}f^{(n+1)}(\xi),\xi\in(a,b).$$

3. Interpolyasiya ko'phadining qurishning Eytken iterasiya usuli:

$$L_{01}(x)=\frac{1}{x_1-x_0}\begin{vmatrix}y_0 & x_0-x\\ y_1 & x_1-x\end{vmatrix}, L_{ii+1}(x)=\frac{1}{x_{i+1}-x_i}\begin{vmatrix}y_i & x_i-x\\ y_{i+1} & x_{i+1}-x\end{vmatrix}, i=0..n-1,$$

$$L_{01..k}(x)=\frac{1}{x_k-x_0}\begin{vmatrix}L_{01...k-1}(x) & x_0-x\\ L_{1..k}(x) & x_k-x\end{vmatrix}, k=1..n; L_n(x)=L_{01..n}(x).$$

6.1-jadval

x_i	y_i	x_i-x	$L_{i-1,i}$	$L_{i-2,i-1,i}$	$L_{i-3,i-2,i-1,i}$	...	$L_{0,1..n}$
x_0	y_0	x_0-x					
x_1	y_1	x_1-x	$L_{0,1}$				
x_2	x_2	x_2-x	$L_{1,2}$	$L_{0,1,2}$			
x_n	y_n	x_n-x	$L_{n-1,n}$	$L_{n-2,n-1,n}$	$L_{i-3,i-2,i-1,i}$	...	$L_{0,1..n}$

Topshiriqlarni Interpolyasiya masalalarini Mathcad dasturida bajarish. Mathcadda Lagranj interpolyasiya formulalarini yaratish mumkin.

Lagranj interpolyasiya ko'phadi. Mathcad dasturida programmasini yozib chiqamiz:

$n:=8$ $f(x):=x*\sin(x)$ $a:=0$ $b:=1$ $h:=(b-a)/n$ *// tugun nuqtalar, funksiya*

$i:=0..n$ $vx_i:=a+ih$ $vy_i=f(vx_i)$ *// interpolyasiya shartlari*

$vx^T=(0\ 0.125\ 0.25\ 0.375\ 0.5\ 0.625\ 0.75\ 0.875\ 1)$

$vy^T=(0\ 0.016\ 0.062\ 0.137\ 0.24\ 0.366\ 0.511\ 0.672\ 0.841)$ *// qiymatlar*

$i:=0..n$ $j:=0..n$ *// indekslarning o'zgarish sohasi*

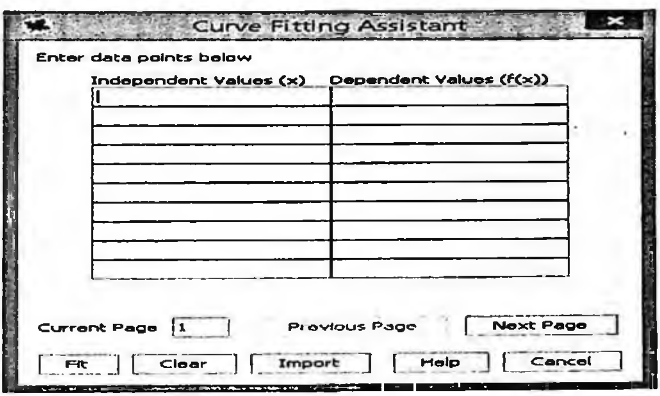
$Ln(x) := \sum_{i=0}^{n} vy_i \prod_{j=0}^{n} if(i=j,1,(x-vx_j)/(vx_i-vx_j))$ // *Lagranj interpolyasiya ko'phadi*

$\overrightarrow{f(t)} = \begin{pmatrix}0.062\\0.24\\0.511\end{pmatrix}$ $\overrightarrow{Ln(t)} = \begin{pmatrix}0.062\\0.24\\0.511\end{pmatrix}$ // *funcsiya va ko'phad qiymatlari*

Topshiriqlarni Interpolyasiya masalalarini Maple dasturida bajarish.

Mapleda ko'phadlar va splaynlar bilan interpolyasiya masalalarini yechish mumkin. Avvalo, with(CurveFitting): komandasi bilan interpolyasiya paketi ochiladi, so'ng PolinomialInterpolation(xdata,ydata,v,opts) (ko'phadli interpolyasiya), Spline(xdata,ydata,v,dgr, endpts) (splayn interpolyasiya) komandalari beriladi. Bu yerda xdata, ydata tugun nuqtalar va interpolyasiyalanuvchi funksiya qimmatlari to'plami: $xdata := [x0, x1, ..., xn], ydata := [y0, y1, ..., yn]$,v:=x yangi interpolyasiya nuqtasi, dgr-splaynning darajasi, opts:=Lagrange, Newton, power-interpolyant turi, endpts:='natural','notacnot','p eriodic'-chegara shartlarning turlari. Bu oynada eng kichik kvadratlar usuli bilan yaqinlashtirish, rasional interpolyasiya (Tile interpolyasiya usuli) larni ham qurish imkoniyati mavjud.

1)Tools>Assistants>CurveFitting komandasini beramiz, jadval chiqadi, uni to'ldiramiz:



6.1-rasm

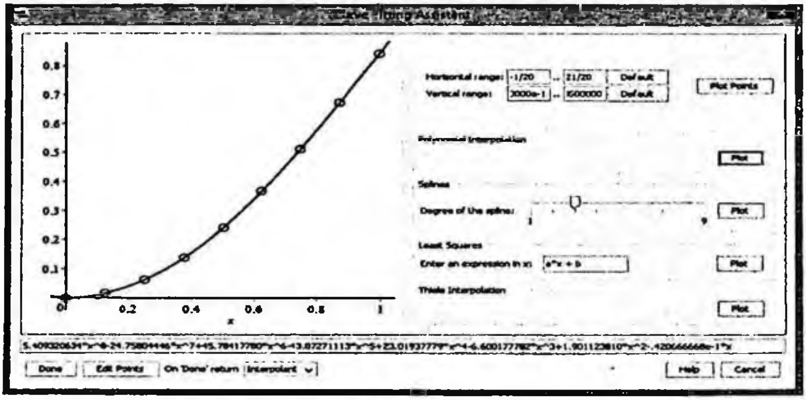
xy:=[[0,0],[0.125,0.016],[0.250,0.062],[0.375,0.137],[0.5,0.24],0.625,0.366],[0.75,0.511],[0.875,0.672],[1,0.841]];

2) Komandalar ro'yxatidan Fit (Fit, Clear, Import, Help,Cancel, Curent Page, Previous, Cancel) komandasini tanlaymiz.

3)Yangi Curve Fitting Assistant oynasidan Polynomial Interpolation komandasidan Plot (chizing) komandasini tanlaymiz.

4) tanlashga muvofiq 8-darajali interpolyasiya ko'phadi chiziladi va javoblar satrida ko'phadning o'zi ko'rinadi:

$$L_8(x) = 5.4093x^8 - 24.7580x^7 + 45.7841x^6 - 43.8727x^5 + 23.0193x^4 - 6.6001x^3 + 1.9011x^2 - 0.0406x$$



6.2-rasm

Individual topshiriqlar.

$y = f(x)$ funksiya $[a,b]$ kesmada berilgan. To'r $x_i = a + ih, i = 0..n, n = 10, h = (b-a)/n$ tuzilib funksiya, Lagranj ko'phadlari bilan interpolyasiyalansin va $\bar{x}_1, \bar{x}_2, \bar{x}_3 \neq x_i$ nuqtalarda qiymatlari solishtirilsin.

6.2-jadval

№	Funksiya	Jvaoblar
1	$f(x) = x^4(1+x^2)^{-1}, [1,2]$	
2	$f(x) = x^2e^{-2x}, [0;1.6]$	
3	$f(x) = x^{-0.5}\ln(x), [1,3]$	
4	$f(x) = x\sin 3(x), [0,1]$	

5	$f(x)=\sqrt{1+x}\lg(1+x),[0.1,1.1]$	
6	$f(x)=x^2\ln(x),[1,2]$	
7	$f(x)=x^2(x^2+1)^{-1},[1,4]$	
8	$f(x)=x\cos(2x),[0,1]$	
9	$f(x)=x^4\ln(x),[1,2]$	
10	$f(x)=\sqrt{x}\ln(x),[1,4]$	
11	$f(x)=x^4(1+x^2)^{-1},[1,2]$	
12	$f(x)=x^3\sqrt{1-x^2},[1,2]$	
13	$f(x)=\sqrt{x}/(1+x),[1,4]$	
14	$f(x)=e^{-x},[1,4]$	
15	$f(x)=xarctg(x),[0,1]$	
16	$f(x)=x\arccos(x),[-0.5,0.5]$	
17	$f(x)=x\arcsin(x),[0,0.9]$	
18	$f(x)=(x+x^3)^{-1},[1,2.2]$	
19	$f(x)=x3^{-x},[0,1.5]$	
20	$f(x)=x^2e^{-x},[0,1]$	
21	$f(x)=x^3/(1+x^2),[0,2]$	
22	$f(x)=(x+x^2)^{-1},[1,3]$	
23	$f(x)=\sqrt{1+x^2},[0,2]$	
24	$f(x)=x^2\sin(x),[0,1]$	
25	$f(x)=x\sin(x),[0,1.6]$	
26	$f(x)=x^3/\sqrt{1+x^2},[-0.4,0.8]$	
27	$f(x)=x^2\cos(x),[0,1]$	
28	$f(x)=x2^{-x},[0,2]$	
29	$f(x)=e^x\sin(x),[0,1.2]$	
30	$f(x)=x^2 arctg(x),[0,1]$	

Nazariy savollar va topshiriqlar.

1. Interpolyasiya masalasi nima?
2. Ko'phad bilan interpolyasiyalash nima?
3. Lagranj interpolyasiya ko'phadi nima?
4. Lagranj interpolyasiya ko'phadining qoldiq hadi nima?

Amaliy mashg'ulot №7

Mavzu: Tengmas oraliqlar uchun Nyuton interpolasion formulasi

Mashg'ulotning maqsadi: Talabalarda tengmas oraliqlar uchun Nyuton interpolasion formulasini yechish ko'nikmasini shakllantirish

Mashg'ulotning jihozi:

a) Ko'rgazmali material

b) Kompyuterning dasturiy ta'minoti

c) Matematik tizimlar

Mashg'ulot texnik vositalari: Pentium IV kompyuterlari (qo'shimcha qurilmalari) bilan jihozlangan kompyuter sinfi

Mashg'ulotning borishi:

a) Auditoriya va talabalarni darsga tayyorligini aniqlash

b) Yo'qlama qilish.

c) Talabalardan o'tilgan mavzularni takrorlash

d) Yangi mavzuning bayoni: kompyuter va ko'rgazmali qurollar yordamida bayon etiladi (Aqliy hujum metodi)

e) Dars yakuni: Amaliy mashg'ulot bo'yicha har bir talabaga individual topshiriqlar beriladi.

Mavzuni qisqacha bayoni

1. Interpolyasiya masalasi: $F(x_i) = y_i$, $i = 0,1,...,n$, $F(x) = ?$.

2.Ko'pxad bilan interpolyasiyalash: $P_n(x_i) = y_i$, $i = 0,1,...,n$, $P_n(x) = ?$.

3.Bo'lingan ayirmalar: $f[x_0,...,x_k]=(f[x_0,...,x_{k-2},x_k]-f[x_0,...,x_{k-1}])/(x_k-x_{k-1}), k\geq 1.$

Ulami quyidagicha qurish maqsadga muvofiq:

6.2-jadval

x_i	$f(x_i)$	$f[x_0,x_i]$	$f[x_0,x_i]$	$f[x_0,x_i]$	...	$f[x_0,x_1,...,x_n]$
x_0	$f(x_0)$	-	-	-	-	-
x_1	$f(x_1)$	$f[x_0,x_1]$	-	-	-	-
x_2	$f(x_2)$	$f[x_0,x_2]$	$f[x_0,x_1,x_2]$	-	-	-
x_3	$f(x_3)$	$f[x_0,x_3]$	$f[x_0,x_1,x_3]$	$f[x_0,x_1,x_2,x_{n-1}]$	-	-
-	-	-	-	-	-	-
x_n	$f(x_n)$	$f[x_0,x_n]$	$f[x_0,x_1,x_n]$	$f[x_0,x_1,x_2,x_n]$	-	$f[x_0,x_1,...,x_n]$

Bo'lingan ayirmalarning quyidagi xossalari mavjud:

1)Bo'lingan ayirmalar o'z argumentlarining ixtiyoriy o'rin almashtirishiga nisbatan simmetrik funksiya;

2)Ularni hisoblash uchun quyidagi formulalar o'rinli:

$$f[x_0,x_1,...,x_k]=\frac{f[x_1,...,x_k]-f[x_0,...,x_{k-1}]}{x_k-x_0}=\frac{f[x_0,...,x_{k-2},x_k]-f[x_0,...,x_{k-1}]}{x_k-x_{k-1}}.$$

Nyuton interpolyasiya formulasini qurish uchun jadvalning bosh diogonali elementlari-bosh bo'lingan ayirmalar kerak. Ular dasturlash tillarida 2 ta rekkurent sikl bilan quriladi:

$$a_0:=f(x_0)$$
$$for\ i:=1\ do\ n$$
$$for\ j=i\ to\ n\ do\ n$$
$$a_j:=(a_j-a_{i-1})/(x_j-x_{i-1})$$

4. Nyuton interpolyasiya formulasi:

$$N_n(f;x)=P_n(x)=f(x_0)+\sum_{k=1}^{n}f[x_0,...,x_k](x-x_0)...(x-x_{k-1}),$$

$$R_n(f,x)=f[x,x_0,...,x_n](x-x_0)..(x-x_n).$$

Topshiriqlarni Interpolyasiya masalalarini Mathcad dasturida bajarish.

Mathcadda Nyuton interpolyasiya formulalarini yaratish mumkin.

Nyuton interpolyasiya ko'phadi.

Mathcad dasturida programmasini yozib chiqamiz:

$n:=8\quad f(x):=x\cdot\sin(x)\quad a:=0\quad b:=1\quad h:=(b-a)/n$ *// tugun nuqtalar, funksiya*

$i:=0..n\quad vx_i:=a+ih\quad vy_i=f(vx_i)$ *// interpolyasiya shartlari*

$vx^T=(0\ \ 0.125\ \ 0.25\ \ 0.375\ \ 0.5\ \ 0.625\ \ 0.75\ \ 0.875\ \ 1)$

$vy^T=(0\ \ 0.016\ \ 0.062\ \ 0.137\ \ 0.24\ \ 0.366\ \ 0.511\ \ 0.672\ \ 0.841)$ *// funksiya qiymatlari*

$a_0:=f(vx_0)$ k:=1..n $a_k:=\sum_{i=0}^{k} vy_i\prod_{j=0}^{k} if(i=j,1,1/(vx_i-vx_j))$ *// bo'lingan ayirmalar*

$Nn(x):=a_0+\sum_{i=1}^{n} a_i\prod_{j=0}^{i-1}(x-vx_j)$ *// Nyuton ko'phadi*

$\overrightarrow{f(t)}=\begin{pmatrix}0.062\\0.24\\0.511\end{pmatrix}\quad \overrightarrow{Nn(t)}=\begin{pmatrix}0.062\\0.24\\0.511\end{pmatrix}$ *// funksiya va ko'phad qiymatlari*

Topshiriqlarni Interpolyasiya masalalarini Mathcad dasturida bajarish.

Maple da ko'phadlar va splaynlar bilan interpolyasiya masalalarini yechish mumkin. Avvalo, with(CurveFitting): komandasi bilan interpolyasiya paketi ochiladi, so'ng PolinomialInterpolation(xdata,ydata,v,opts) (ko'phadli interpolyasiya), Spline(xdata, ydata, v, dgr, endpts) (splayn interpolyasiya) komandalari beriladi. Bu yerda xdata, ydata tugun nuqtalar va interpolyasiyalanuvchi funksiya qimmatlari to'plami: $xdata:=[x0,x1,...,xn]$, $ydata:=[y0,y1,...,yn]$,v:=x interpolyasiya nuqtasi, dgr-splaynning darajasi, opts:=Lagrange, Newton, power-interpolyant turi.

1)Tools>Assistants>CurveFitting komandasini beramiz, jadval chiqadi va uni to'ldiramiz:

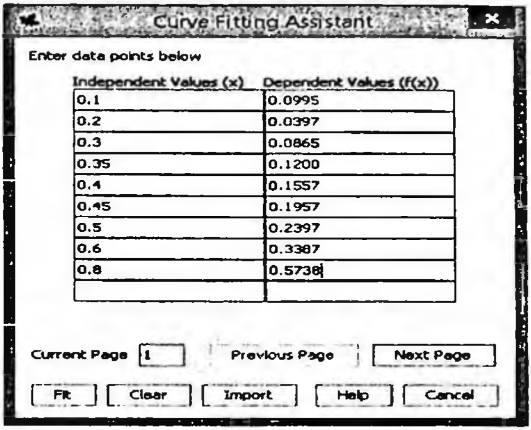
$xy:=[[0,0],[0.125,0.016],[0.250,0.062],[0.375,0.137],[0.5,0.24],0.625,0.366],[0.75,0.511],[0.875,0.672],[1,0.841]]);$

2) Komandalar ro'yxatidan Fit (Fit, Clear, Import, Help, Cancel, Curent Page, Previous, Cancel) komandasini tanlaymiz.

3)Yangi Curve Fitting Assistant oynasidan Polynomial Interpolation komandasidan Plot (chizing) komandasini tanlaymiz.

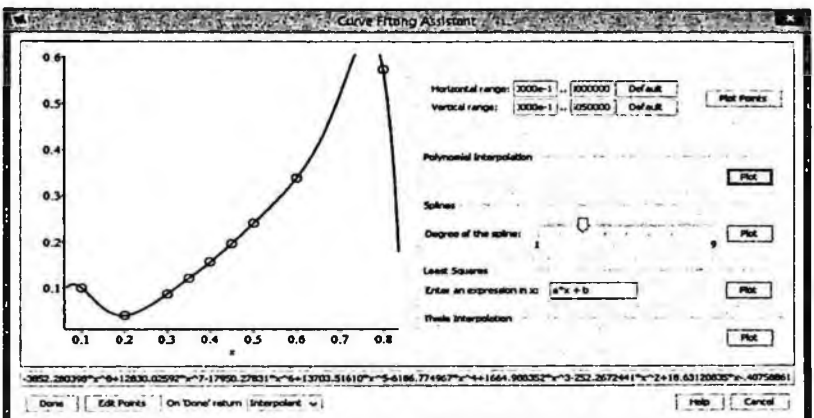
4) Tanlashga muvofiq 8-darajali interpolyasiya ko'phadi chiziladi va javoblar satrida ko'phadning o'zi ko'rinadi:

$$N_8(x) = 3852.2803x^8 - 1283.0159x^7 - 1795.2783x^6 + 1370.5761x^5 - 6186.7749x^4 + 1664.9883x^3 - 252.2672x^2 + 1865.12x - 0.4075$$



7.1-rasm

Ko'phadning koeffisientlari ancha katta sonlar. Bu hisoblashlarning turg'un emasligini ko'rsatadi.



7.2-rasm

Individual topshiriqlar.

$y = f(x)$ funksiya $[a,b]$ kesmada berilgan. To'r $x_i = a + ih, i = 0..n, n = 10, h = (b-a)/n$ tuzilib funksiya Nyuton interpolyasiya ko'phadlari bilan interpolyasiyalansin va $\bar{x}_1, \bar{x}_2, \neq x_i$ nuqtalarda qiymatlari solishtirilsin. Topshiriqlar jadvali oldingi bobdan olinsin.

7.1-jadval

№	Funksiya	Jvaoblar
1	$f(x)=x^4(1+x^2)^{-1}$, [1,2]	
2	$f(x)=x^2e^{-2x}$, [0;1.6]	
3	$f(x)=x^{-0.5}\ln(x)$, [1,3]	
4	$f(x)=x\sin 3(x)$,[0,1]	
5	$f(x)=\sqrt{1+x}\lg(1+x)$,[0.1,1.1]	
6	$f(x)=x^2\ln(x)$,[1,2]	
7	$f(x)=x^2(x^2+1)^{-1}$,[1,4]	
8	$f(x)=x\cos(2x)$,[0,1]	
9	$f(x)=x^4\ln(x)$,[1,2]	
10	$f(x)=\sqrt{x}\ln(x)$,[1,4]	
11	$f(x)=x^4(1+x^2)^{-1}$,[1,2]	
12	$f(x)=x^3\sqrt{1-x^2}$,[1,2]	
13	$f(x)=\sqrt{x}/(1+x)$,[1,4]	
14	$f(x)=e^{-x}$,[1,4]	
15	$f(x)=xarctg(x)$,[0,1]	
16	$f(x)=x\arccos(x)$,[−0.5,0.5]	
17	$f(x)=x\arcsin(x)$,[0,0.9]	
18	$f(x)=(x+x^3)^{-1}$,[1,2.2]	
19	$f(x)=x3^{-x}$,[0,1.5]	
20	$f(x)=x^2e^{-x}$,[0,1]	
21	$f(x)=x^3/(1+x^2)$,[0,2]	
22	$f(x)=(x+x^2)^{-1}$,[1,3]	
23	$f(x)=\sqrt{1+x^2}$,[0,2]	

24	$f(x)=x^2\sin(x),[0,1]$	
25	$f(x)=x\sin(x),[0,1.6]$	
26	$f(x)=x^3/\sqrt{1+x^2},[-0.4,0.8]$	
27	$f(x)=x^2\cos(x),[0,1]$	
28	$f(x)=x2^{-x},[0,2]$	
29	$f(x)=e^x\sin(x),[0,1.2]$	
30	$f(x)=x^2\operatorname{arctg}(x),[0,1]$	

Nazariy savollar va topshiriqlar.

1.Interpolyasiya masalasi nima?

2.Ko'phad bilan interpolyasiyalash nima?

3. Bo'lingan ayirmalar nima?

4. Nyuton interpolyasiya ko'phadi va qoldiq hadi nima ?

5. Lagranj interpolyasiya ko'phadi va qoldiq hadi nima?

Amaliy mashg'ulot №8

Mavzu: Teng oraliqlar uchun Nyuton interpolasion formulasi

Mashg'ulotning maqsadi: Talabalarda teng oraliqlar uchun Nyuton interpolasion formulasini yechish ko'nikmasini shakllantirish

Mashg'ulotning jihozi:

a) Ko'rgazmali material

b) Kompyuterning dasturiy ta'minoti

c) Matematik tizimlar

Mashg'ulot texnik vositalari: Pentium IV kompyuterlari (qo'shimcha qurilmalari) bilan jihozlangan kompyuter sinfi

Mashg'ulotning borishi:

a) Auditoriya va talabalarni darsga tayyorligini aniqlash

b) Yo'qlama qilish.

c) Talabalardan o'tilgan mavzularni takrorlash

d) Yangi mavzuning bayoni: kompyuter va ko'rgazmali qurollar yordamida bayon etiladi (Aqliy hujum metodi)

e) Dars yakuni: Amaliy mashg'ulot bo'yicha har bir talabaga individual topshiriqlar beriladi.

Mavzuni qisqacha bayoni

1. Chekli ayirmalar. [a, b] kesmada tekis to'r $a = x_0 < x_1 < \cdots < x_n = b$ - bir-biridan teng uzoqlikda yotgan $x_i = a + ih, i = 0..n, h = (b - n)/n$ nuqtalar to'plamini tuzamiz. Bu nuqtalarda $y_i = f(x_i), i = 0..n$ qiymatlarda berilgan bo'lsin. Quyidagi chekli ayirmalarni kiritamiz:

$\Delta y_i = y_{i+1} - y_i,$ birinchi tartibli

$\Delta^2 y_i = \Delta y_{i+1} - \Delta y_i = y_{i+1} - 2y_i + y_{i-1},$ ikkinchi tartibli,

....

$\Delta^k y_i = \Delta^{k-1} y_{i+1} - \Delta^{k-1} y_i = \sum_{j=0}^{k} (-1)^j C_k^j \; y_{k+i-j},$ k- tartibli.

2. Nyutonning birinchi interpolasion formulasi. Chekli ayirmalar formulasini Nyutonning birinchi formulasiga qo'yamiz:

$$N_n(f;x) = P_n(x) = f(x_0) + \sum_{k=1}^{n} f[x_0,\dots,x_k](x-x_0)\dots(x-x_{k-1}).$$

$$R_n(f,x) = f[x,x_0,\dots,x_n](x-x_0)\dots(x-x_n).$$

Ravshanki,

$$f[x_0,x_1] = \frac{1}{h}(f_1 - f_0) = \frac{\Delta f_0}{h},$$

$$f[x_0,x_1,x_2] = \frac{1}{x_2 - x_0}[f[x_1,x_2] - f[x_0,x_1]] = \frac{1}{2h}[\Delta f_1 - \Delta f_0] = \frac{1}{2!h^2}\Delta^2 f_0.$$

......

$$f[x_0,x_1,\dots x_k] = \frac{1}{x_k - x_0}[f[x_1,\dots,x_k] - f[x_0,x_{k-1}]] = \frac{1}{kh}[\frac{1}{(k-1)!h^{k-1}}\Delta^{k-1} f_1 - \frac{1}{(k-1)!h^{k-1}}\Delta^{k-1} f_0] = \frac{1}{k!h^k}\Delta^k f_0$$

Demak, biz quyidagi formulalarga egamiz:

$$a_0 = f(x_0), a_k = f[x_0,\dots,x_k] = \frac{1}{k!h^k}\Delta^k f_0, k = 1..n.$$

va Nyutonning birinchi interpolasion formulasi quyidagicha yoziladi:

$$N_n(x) = f_0 + \frac{\Delta f_0}{h}(x-x_0) + \frac{\Delta^2 f_0}{2!h^2}(x-x_0)(x-x_1) + \dots + \frac{\Delta^n f_0}{n!h^n}(x-x_0)\dots(x-x_{n-1})$$

Bu yerda $x = x_0 + qh$ almashtirish bajarsak quyidagi formulaga kelamiz:

$$N_n(x) = f_0 + \frac{q}{1!}\Delta f_0 + \frac{q(q-1)}{2!}\Delta^2 f_0 + .. + \frac{q(q-1)..(q-n+1)}{n!}\Delta^n f_0.$$

3. Xuddi shunday, Nyutonning ikkinchi interpolasion formulasi

$$N_n(f;x) = P_n(x) = a_0 + \sum_{k=1}^{n} a_k (x - x_n)...(x - x_{n-k+1})$$

quyidagi ko'rinishni oladi:

$$a_0 = f(x_n), a_k = \frac{1}{k!h^k}\Delta^k f_{n-k}, k = 1..n$$

$$N_n(x) = f_n + \frac{\Delta f_{n-1}}{h}(x - x_n) + \frac{\Delta^2 f_{n-2}}{2!h^2}(x - x_n)(x - x_{n-1}) + .. + \frac{\Delta^n f_0}{n!h^n}(x - x_n)..(x - x_1)$$

Bu yerda $x = x_n + qh$ almashtirish bajarsak quyidagi formulaga kelamiz:

$$N_n(x) = f_n + \frac{q}{1!}\Delta f_{n-1} + \frac{q(q+1)}{2!}\Delta^2 f_{n-2} + .. + \frac{q(q+1)..(q+n-1)}{n!}\Delta^n f_0.$$

Qoldiq hadning ko'rinishi esa mos ravishda quyidagicha bo'ladi

$$R_n(q) = h^{n+1}\frac{q(q-1)...(q-n)}{(n+1)!}f^{(n+1)}(\xi).$$

$$R_n(q) = h^{n+1}\frac{q(q+1)...(q+n)}{(n+1)!}f^{(n+1)}(\eta)$$

Topshiriqlarni Interpolyasiya masalalarini Mathcad dasturida bajarish. Mathcadda Nyuton, Lagranj interpolyasiya formulalarini teng uzoqlikda joylashgan nuqtalar uchun yaratish mumkin.

Nyuton interpolyasiya ko'phadi. Mathcad dasturida programmani yozib chiqamiz:

$n := 8 \quad f(x) := x * \sin(x) \quad a := 0 \quad b := 1 \quad h := (b-a)/n$ *// tugun nuqtalar, funksiya*

$i := 0..n \quad vx_i := a + ih \quad vy_i = f(vx_i)$ *// interpolyasiya shartlari*

$vx^T = (0 \ \ 0.125 \ \ 0.25 \ \ 0.375 \ \ 0.5 \ \ 0.625 \ \ 0.75 \ \ 0.875 \ \ 1)$

$vy^T = (0 \ \ 0.016 \ \ 0.062 \ \ 0.137 \ \ 0.24 \ \ 0.366 \ \ 0.511 \ \ 0.672 \ \ 0.841)$ *// funksiya qiymatlari*

$a_0 := f(vx_0)$ k:=1..n $\quad a_k := \sum_{i=0}^{k} vy_i \prod_{j=0}^{k} if(i = j, 1, 1/(vx_i - vx_j))$ *// bo'lingan ayirmalar*

$Nn(x) := a_0 + \sum_{i=1}^{n} a_i \prod_{j=0}^{i-1}(x - vx_j)$ *// Nyuton ko'phadi*

$$\overrightarrow{f(t)} = \begin{pmatrix} 0.062 \\ 0.24 \\ 0.511 \end{pmatrix} \quad \overrightarrow{Nn(t)} = \begin{pmatrix} 0.062 \\ 0.24 \\ 0.511 \end{pmatrix}$$ *// natijalar*

Interpolyasiya masalalarini Maple dasturida bajarish.

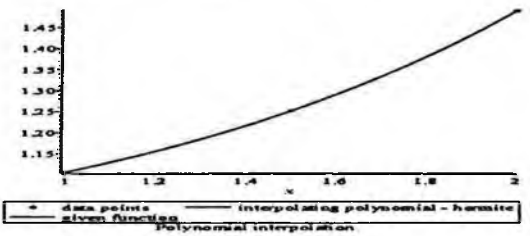
Maple da ko'phadlar va splaynlar bilan interpolyasiya masalalarini yechish mumkin. Avvalo, with(CurveFitting): komandasi bilan interpolyasiya paketi ochiladi, so'ngra PolinomialInterpolation(xdata,ydata,v,opts) (ko'phadli interpolyasiya), Spline(xdata,ydata,v,dgr, endpts) (splayn interpolyasiya) komandalari beriladi. Bu yerda xdata, ydata tugun nuqtalar va interpolyasiyalanuvchi funksiya qimmatlari to'plami: $xdata := [x0, x1, ..., xn], ydata := [y0, y1, ..., yn]$,v:=x interpolyasiya nuqtasi, dgr-splaynning darajasi, opts:=Lagrange, Newton,power-interpolyant turi, endpts:='natural','notacnot','p yeriodic'-chegara shartlarning turlari.

with(Student[NumericalAnalysis]):

$xy := [[0,1],[1,2],[2,9],[3,28]];$

N:= PolinomialInterpolation(xy, independentvar=x, method=newton);

expand(Int yerpolant(N)); // $1+(2/3)x-x^2+(4/3)x^3$



8.1-rasm

Individual topshiriqlar.

$y = f(x)$ funksiya $[a,b]$ kesmada berilgan. To'r $x_i = a + ih, i = 0..n, n = 10, h = (b-a)/n$ tuzilib funksiya Nyuton bilan interpolyasiyalansin va $\bar{x}_1, \bar{x}_2, \bar{x}_3 \neq x_i$ nuqtalarda qiymatlari solishtirilsin.

8.1-jadval

№	Funksiya	Jvaoblar
1	$f(x)=x^4(1+x^2)^{-1},[1,2]$	
2	$f(x)=x^2e^{-2x},[0;1.6]$	
3	$f(x)=x^{-0.5}\ln(x),[1,3]$	
4	$f(x)=x\sin 3(x),[0,1]$	
5	$f(x)=\sqrt{1+x}\lg(1+x),[0.1,1.1]$	
6	$f(x)=x^2\ln(x),[1,2]$	
7	$f(x)=x^2(x^2+1)^{-1},[1,4]$	
8	$f(x)=x\cos(2x),[0,1]$	
9	$f(x)=x^4\ln(x),[1,2]$	
10	$f(x)=\sqrt{x}\ln(x),[1,4]$	
11	$f(x)=x^4(1+x^2)^{-1},[1,2]$	
12	$f(x)=x^3\sqrt{1-x^2},[1,2]$	
13	$f(x)=\sqrt{x}/(1+x),[1,4]$	
14	$f(x)=e^{-x},[1,4]$	
15	$f(x)=x\,arctg(x),[0,1]$	
16	$f(x)=x\arccos(x),[-0.5,0.5]$	
17	$f(x)=x\arcsin(x),[0,0.9]$	
18	$f(x)=(x+x^3)^{-1},[1,2.2]$	
19	$f(x)=x3^{-x},[0,1.5]$	
20	$f(x)=x^2e^{-x},[0,1]$	
21	$f(x)=x^3/(1+x^2),[0,2]$	
22	$f(x)=(x+x^2)^{-1},[1,3]$	
23	$f(x)=\sqrt{1+x^2},[0,2]$	
24	$f(x)=x^2\sin(x),[0,1]$	
25	$f(x)=x\sin(x),[0,1.6]$	

26	$f(x)=x^3/\sqrt{1+x^2},[-0.4,0.8]$	
27	$f(x)=x^2\cos(x),[0,1]$	
28	$f(x)=x2^{-x},[0.2]$	
29	$f(x)=e^x\sin(x),[0,1.2]$	
30	$f(x)=x^2\,\mathrm{arc}tg(x),[0,1]$	

Nazariy savollar va topshiriqlar.

1. Interpolyasiya masalasi nima?
2. Ko'phad bilan interpolyasiyalash nima?
3. Chekli ayirmalar nima?
4. Birinchi Nyuton interpolyasiya ko'phadi nima ?
5. Ikkinchi Nyuton interpolyasiya ko'phadi nima ?

Amaliy mashg'ulot №9

Mavzu: Splaynlar bilan yaqinlashtirish

Mashg'ulotning maqsadi: Talabalarda splaynlar bilan yaqinlashtirish ko'nikmasini shakllantirish

Mashg'ulotning jihozi:

a) Ko'rgazmali material

b) Kompyuterning dasturiy ta'minoti

c) Matematik tizimlar

Mashg'ulot texnik vositalari: Pentium IV kompyuterlari (qo'shimcha qurilmalari) bilan jihozlangan kompyuter sinfi

Mashg'ulotning borishi:

a) Auditoriya va talabalarni darsga tayyorligini aniqlash

b) Yo'qlama qilish.

c) Talabalardan o'tilgan mavzularni takrorlash

d) Yangi mavzuning bayoni: kompyuter va ko'rgazmali qurollar yordamida bayon etiladi (Aqliy hujum metodi)

e) Dars yakuni: Amaliy mashg'ulot bo'yicha har bir talabaga individual topshiriqlar beriladi.

Mavzuni qisqacha bayoni

1. Chiziqli interpolyasiya splayni $S(x) = S_1(x) \in C[a,b]$ - har bir kesmacha $[x_i, x_{i+1}]$ da chiziqli funksiya va berilgan nuqtalarda berilgan qiymatlarni qabul qiladi: $S(x_i) = f(x_i)$, $i = 0 \dots n$, yani

$$S_1(x) = f(x_i) + f[x_i, x_{i+1}](x - x_i), x_i \le x \le x_{i+1} \quad , |f(x) - S_1(x)| \le \frac{M_2 h^2}{8}, M_2 = \max_{x \in [a,b]} |f''(x)|.$$

2. Kubik interpolyasiya splayni $S(x) = S_3(x) \in S^2[a,b]$ har bir kesmacha $[x_i, x_{i+1}]$ da uchinchi darajali ko'pxad va berilgan nuqtalarda berilgan qiymatlarni qabul qiladi va ikkita chegara shartga bo'ysinadi:

$S_3(x_i) = f(x_i)$, $i = 0 \dots n$, yani

$$S(f;x) = \frac{M_i(x_{i+1} - x)^3}{6h_i} + \frac{M_{i+1}(x - x_i)^3}{6h_i} + (f_i - \frac{M_i h_i^2}{6})\frac{x_{i+1} - x}{h_i} + (f_{i+1} - \frac{M_{i+1} h_i^2}{6})\frac{x - x_i}{h_i},$$

$$2M_0 + \beta_0 M_1 = d_0, \; \alpha_i M_{i-1} + 2M_i + \beta_i M_{i+1} = d_i, \; i = 1,2,\dots,n-1 \;, \alpha_n M_0 + 2M_1 = d_n$$

3. Interpolyasiya splaynlarining ekstremal xossalari:

$$\int_a^b [S^{(2)}(x)]^2 dx = \min_{f(x_i) = z_i, i=0..n} \int_a^b [f^{(2)}(x)]^2 dx,$$

$$\int_a^b [f^{(2)}(x) - S^{(2)}(x)]^2 dx = \min_{f(x_i) = g(x_i), i=0..n} \int_a^b [f^{(2)}(x) - g^{(2)}(x)]^2 dx.$$

Oxirgi ekstremal xossadan kelib chiqadi: kvadrati bilan integrallanuvchi funksiyalar sinfida kubik interpolyasiya splaynining qoldiq hadi eng kichkina. Bu xossa barcha toq darajali interpolyasya splaynlari uchun o'rinli.

Mathcadda interpolyasiya masalalarini yechish.

Mathcadda splayn interpolyasiya formulalarini ichki funksiyalar yordamida qurish mumkin. Masalan, chiziqli interpolyasiya splayni $S1(x) := \mathrm{linterp}(vx, vy, x)$ komandasi bilan quriladi. Kubik splayn esa ikkita ichki funksiya bilan quriladi. Avval, $M := \mathrm{cspline}(vx, vy)$ komandasi bilan funksiya jadvali asosida splaynning koeffisientlari topiladi. So'ng, ular asosida $s3(x) := \mathrm{linterp}(M, vx, vy, x)$ ichki funksiya yordamida natural kubik splayning qiymati hisoblanadi. Yana boshqa chegara shartli splaynlar ham mavjud.

Splaynlar bilan interpolyasiyalash:

$S_1(x)=f(x_i)+f[x_i,x_{i+1}](x-x_i), x_i \le x \le x_{i+1}$, -chiziqli interpolyasiya splayni.

$$S_3(f;t)=\frac{M_i(x_{i+1}-t)^3}{6h_i}+\frac{M_{i+1}(t-x_i)^3}{6h_i}+(f_i-\frac{M_ih_i^2}{6})\frac{x_{i+1}-t}{h_i}+(f_{i+1}-\frac{M_{i+1}h_i^2}{6})\frac{t-x_i}{h_i},$$

$x_i \le x \le x_{i+1}, i=0..n-1$,-kubik interpolyasiya splayni.

Chiziqli S1, kubik S3 splaynlar quyidagicha quriladi:

$n:=8 \quad f(x):=x*\sin(x) \quad a:=0 \quad b:=1 \quad h:=(b-a)/n$ *// tugun nuqtalar, funksiya*

$i:=0..n \quad vx_i:=a+ih \quad vy_i=f(vx_i)$ *// interpolyasiya shartlari*

$vx^T=(0\ 0.125\ 0.25\ 0.375\ 0.5\ 0.625\ 0.75\ 0.875\ 1)$

$vy^T=(0\ 0.016\ 0.062\ 0.137\ 0.24\ 0.366\ 0.511\ 0.672\ 0.841)$ *// qiymatlar*

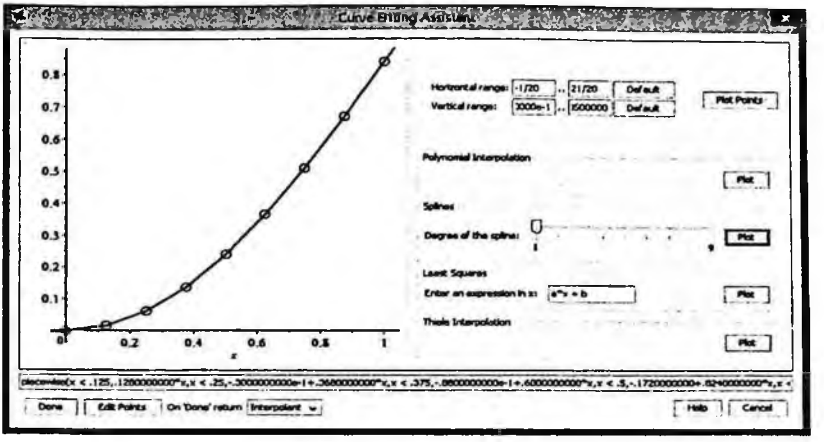
$S1(x):=l\,\mathrm{interp}(vx,vy,x) \quad M:=cspline(vx,vy) \quad S3(x):=\mathrm{interp}(M,vx,vy,x)$ *//splaynlar*

$\overrightarrow{f(t)}=\begin{pmatrix}0.062\\0.24\\0.511\end{pmatrix} \quad \overrightarrow{s1(t)}=\begin{pmatrix}0.062\\0.24\\0.511\end{pmatrix} \quad \overrightarrow{s3(t)}=\begin{pmatrix}0.062\\0.24\\0.511\end{pmatrix}$ *// qiymatlar*

Mapleda interpolyasiya masalalarini yechish.

Mapleda ko'phadlar va splaynlar bilan interpolyasiya masalalarini yechish mumkin. Avvalo, with(CurveFitting): komandasi bilan interpolyasiya paketi ochiladi, so'ng PolinomialInterpolation(xdata,ydata,v,opts) (ko'phadli interpolyasiya), Spline(xdata, ydata,v, dgr, endpts) (splayn interpolyasiya) komandalari beriladi. Bu yerda xdata, ydata tugun nuqtalar va interpolyasiyalanuvchi funksiya qimmatlari to'plami: $xdata:=[x0,x1,...,xn], ydata:=[y0,y1,...,yn]$, v:=x interpolyasiya nuqtasi, dgr-splaynning darajasi, opts:=Lagrange, Newton, power-interpolyant turi, endpts:='natural','notacnot','p yeriodic'-chegara shartlarning turlari.

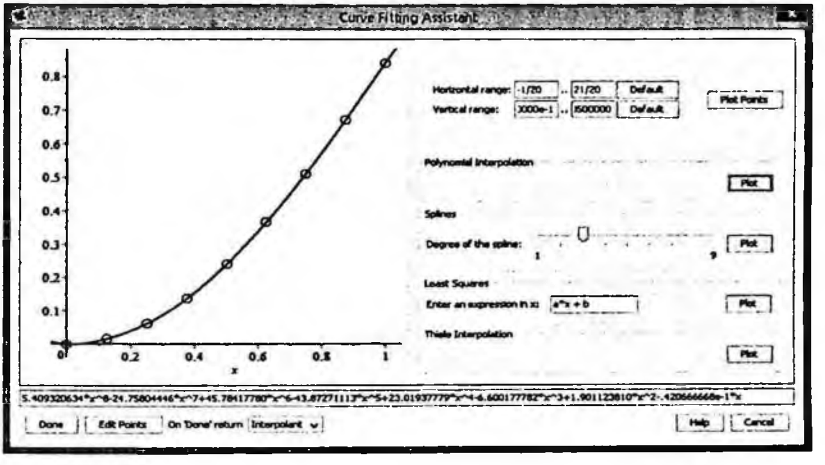
```
xy:=[[0,0],[0.125,0.016],[0.250,0.062],[0.375,0.137],[0.5,0.24],0.625,0.366],[0.75,0.511],[0.875,0.672],[1,0.841]]);
S1:=LinearSpline(xy, independentvar=x);
```



9.1-rasm

Degree=1.

S3:=CubicSpline(xy, independentvar=x);



9.2-rasm

Individual topshiriqlar.

$y=f(x)$ funksiya $[a,b]$ kesmada berilgan. To'r $x_i=a+ih, i=0..n, n=10, h=(b-a)/n$ tuzilib funksiya splaynlar bilan interpolyasiyalansin va $\bar{x}_1,\bar{x}_2,\bar{x}_3 \neq x_i$ nuqtalarda qiymatlari solishtirilsin.

9.1-jadval

№	Funksiya	Jvaoblar
1	$f(x)=x^4(1+x^2)^{-1},[1,2]$	
2	$f(x)=x^2e^{-2x},[0;1.6]$	
3	$f(x)=x^{-0.5}\ln(x),[1,3]$	
4	$f(x)=x\sin 3(x),[0,1]$	
5	$f(x)=\sqrt{1+x}\lg(1+x),[0.1,1.1]$	
6	$f(x)=x^2\ln(x),[1,2]$	
7	$f(x)=x^2(x^2+1)^{-1},[1,4]$	
8	$f(x)=x\cos(2x),[0,1]$	
9	$f(x)=x^4\ln(x),[1,2]$	
10	$f(x)=\sqrt{x}\ln(x),[1,4]$	
11	$f(x)=x^4(1+x^2)^{-1},[1,2]$	
12	$f(x)=x^3\sqrt{1-x^2},[1,2]$	
13	$f(x)=\sqrt{x}/(1+x),[1,4]$	
14	$f(x)=e^{-x},[1,4]$	
15	$f(x)=xarctg(x),[0,1]$	
16	$f(x)=x\arccos(x),[-0.5,0.5]$	
17	$f(x)=x\arcsin(x),[0,0.9]$	
18	$f(x)=(x+x^3)^{-1},[1,2.2]$	
19	$f(x)=x3^{-x},[0,1.5]$	
20	$f(x)=x^2e^{-x},[0,1]$	
21	$f(x)=x^3/(1+x^2),[0,2]$	
22	$f(x)=(x+x^2)^{-1},[1,3]$	
23	$f(x)=\sqrt{1+x^2},[0,2]$	
24	$f(x)=x^2\sin(x),[0,1]$	

25	$f(x)=x\sin(x),[0,1.6]$	
26	$f(x)=x^3/\sqrt{1+x^2},[-0.4,0.8]$	
27	$f(x)=x^2\cos(x),[0,1]$	
28	$f(x)=x2^{-x},[0,2]$	
29	$f(x)=e^x\sin(x),[0,1.2]$	
30	$f(x)=x^2\operatorname{arctg}(x),[0,1]$	

Nazariy savollar va topshiriqlar.

1. Ko'phadli interpolyasiya splaynlari nima?
2. Chiziqli interpolyasiya splayni nima?
3. Kubik interpolyasiya splayni nima?
4. Chiziqli interpolyasiya splaynining qoldig'i nima?
5. Chiziqli interpolyasiya splaynining ekstremal xossalari nima nima?
6. Kubik interpolyasiya splaynining ekstremal xossalari nima?

Amaliy mashg'ulot №10

Mavzu: Integrallarni taqribiy hisoblashda trapesiya formulasi

Mashg'ulotning maqsadi: Talabalarda integrallarni taqribiy hisoblashda trapesiya formulasi ko'nikmasini shakllantirish

Mashg'ulotning jihozi:

a) Ko'rgazmali material

b) Kompyuterning dasturiy ta'minoti

c) Matematik tizimlar

Mashg'ulot texnik vositalari: Pentium IV kompyuterlari (qo'shimcha qurilmalari) bilan jihozlangan kompyuter sinfi

Mashg'ulotning borishi:

a) Auditoriya va talabalarni darsga tayyorligini aniqlash

b) Yo'qlama qilish.

c) Talabalardan o'tilgan mavzularni takrorlash

d) Yangi mavzuning bayoni: kompyuter va ko'rgazmali qurollar yordamida bayon etiladi (Muammoli o'qitish texnologiyasi)

e) Dars yakuni: Amaliy mashg'ulot bo'yicha har bir talabaga individual topshiriqlar beriladi.

Mavzuni qisqacha bayoni

1.To'g'ri to'rtburchaklar formulasi:

$$J_h^{TT}(f)=h\sum_{i=0}^{n-1}f(x_i+h/2),\; R_h^{TT}(f)=J(f)-J_h^{TT}(f)=\frac{h^2(b-a)}{24}f''(c).$$

Usulning chizmasi quyida rasm 1 da ko'rsatilgan.

2.Trapesiya formulasi:

$$J_h^{TT}(f)=\frac{h}{2}[f(x_0)+2(f(x_1)+...+f(x_{n-1}))+f(x_n)],\; R_h^{TT}(f)=-\frac{h^2(b-a)}{12}f''(c)$$

Usulning chizmasi quyida rasm 2 da ko'rsatilgan.

Topshiriqni Mathcadda bajarish.

1. To'g'ri to'rtburchaklar formulasi:

$f(x):=\sin(x)\ a:=0\ b:=\pi\ n:=20$ // *funksiya, oraliq, bo'linishlar soni*

$J:=\int_a^b f(x)dx\ \ J=2$ // *hisoblanadigan integral*

$h:=(b-a)\ \ i:=0..n\ \ vx_i:=a+i*h$ // *qadam, tugun nuqtalar*

$J_h^{TT}(f)=h\sum_{i=0}^{n-1}f(x_i+h/2)\ \ J=1.99$ // *TT formulasi va qiymati*

2.Trapesiya formulasi:

$f(x):=\sin(x)\ a:=0\ b:=\pi\ n:=20$ // *funksiya, oraliq, bo'linishlar soni*

$J:=\int_a^b f(x)dx\ \ J=2$ // *hisoblanadigan integral*

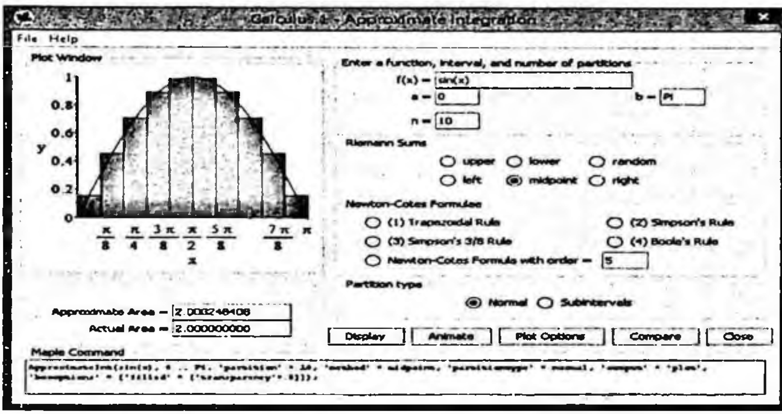
$h:=(b-a)\ \ i:=0..n\ \ vx_i:=a+i*h$ // *qadam, tugun nuqtalar*

$J_h^{T}(f)=\frac{h}{2}\sum_{i=0}^{n-1}[f(x_i)+f(x_{i+1})],\ J=1.996$ //*Trapesiya formulasi, qiymati*

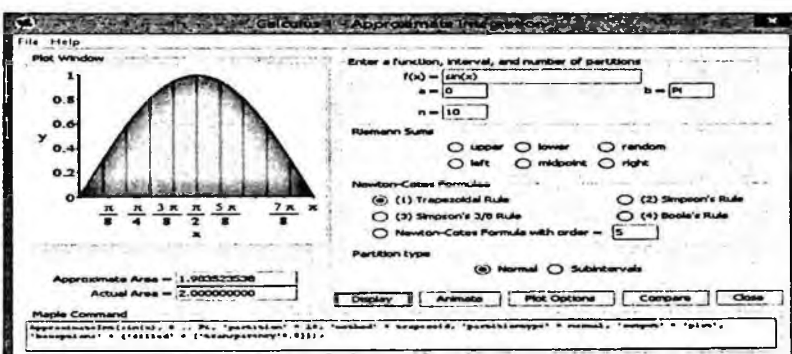
Izoh. Barcha algoritmlarni birlashtiring. (bu juda oson, chunki algoritmlarda formulalar satrgina farqli xolos).

Topshiriqlarni Mapleda bajarish.

Mapleda integrallarni $\mathrm{int}(f(x), x = a..b)$ komandalar bilan bevosita hisoblash mumkin. Lekin, Maple da integrallarni interaktiv usulda ham hisoblash mumkin. Buning uchun Tools>Tutors>Calculus>Single-Variable>Approximate Integration komandasini beramiz: Bu yerda integrallash uchun funksiya kiritish, chegaralarni berish, kesmadagi oraliqlar sonini berish, Riman yig'indisini hisoblash uchun tugun nuqtalar tanlash (6 xil usul mavjud), taqribiy integrallash usullarini tanlash (5 xil usul), integralning aniq va taqribiy qiymatlarini ko'rish va animasiyani ko'rish imkoniyatlari mavjud.



10.1-rasm



10.2-rasm

Interaktiv oynada aniqlikning kamayish tartibida Nyuton-Kotes, Simpson, to'g'rito'rtburchaklar, trapesiyalar formulalari bilan hisoblashlar keltirilgan.

Individual topshiriqlar

$y=f(x)$ funksiya, $[a,b]$ kesma berilgan. To'r $x_i=a+ih, i=0..n, n=10, h=(b-a)/n$ tuzilib funksiya, trapesiya, to'g'rito'rtburchaklar formulasi bilan taqribiy integrallansin. Xatoliklar solishtirilsin. Nuqtalar soni o'zgartirilib xatolikning kamayishi kuzatilsin:

10.1-jadval

№	Funksiya	javoblar
1	$f(x)=x^4(1+x^2)^{-1}$, [1,2]	
2	$f(x)=x^2e^{-2x}$, [0;1.6]	
3	$f(x)=x^{-0.5}\ln(x)$, [1,3]	
4	$f(x)=x\sin 3(x)$, [0,1]	
5	$f(x)=\sqrt{1+x}\lg(1+x)$,[0.1,1.1]	
6	$f(x)=x^2\ln(x)$, [1,2]	
7	$f(x)=x^2(x^2+1)^{-1}$, [1,4]	
8	$f(x)=x\cos(2x)$,[0,1]	
9	$f(x)=x^4\ln(x)$,[1,2]	
10	$f(x)=\sqrt{x}\ln(x)$,[1,4]	
11	$f(x)=x^4(1+x^2)^{-1}$,[1,2]	
12	$f(x)=x^3\sqrt{1-x^2}$,[1,2]	
13	$f(x)=\sqrt{x}/(1+x)$,[1,4]	
14	$f(x)=e^{-x}$,[1,4]	
15	$f(x)=x\,arctg(x)$,[0,1]	
16	$f(x)=x\arccos(x)$,[-0.5,0.5]	
17	$f(x)=x\arcsin(x)$,[0,0.9]	
18	$f(x)=(x+x^3)^{-1}$,[1,2.2]	

19	$f(x)=x3^{-x},[0,1.5]$	
20	$f(x)=x^2e^{-x},[0,1]$	
21	$f(x)=x^3/(1+x^2),[0,2]$	
22	$f(x)=(x+x^2)^{-1},[1,3]$	
23	$f(x)=\sqrt{1+x^2},[0,2]$	
24	$f(x)=x^2\sin(x),[0,1]$	
25	$f(x)=x\sin(x),[0,1.6]$	
26	$f(x)=x^3/\sqrt{1+x^2},[-0.4,0.8]$	
27	$f(x)=x^2\cos(x),[0,1]$	
28	$f(x)=x2^{-x},[0,2]$	
29	$f(x)=e^x\sin(x),[0,1.2]$	
30	$f(x)=x^2\operatorname{arc}tg(x),[0,1]$	

Nazariy savollar va topshiriqlar.

1. Taqribiy integrallash (kvadratura) formulasi nima?
2. Kvadratura formulasining koeffisientlari, tugun nuqtalari nima?
3. Trapesiya kvadratura formulasining qoldiq hadlari nima?
4. To'g'ri to'rtburchaklar kvadratura formulasining qoldiq hadlari nima?
5. Kadratura formulasining qoldiq hadlari nima?

Amaliy mashg'ulot №11

Mavzu: Integrallarni taqribiy hisoblashda Simpson formulasi

Mashg'ulotning maqsadi: Talabalarda integrallarni taqribiy hisoblashda simpson formulasi ko'nikmasini shakllantirish

Mashg'ulotning jihozi:

a) Ko'rgazmali material

b) Kompyuterning dasturiy ta'minoti

c) Matematik tizimlar

Mashg'ulot texnik vositalari: Pentium IV kompyuterlari (qo'shimcha qurilmalari) bilan jihozlangan kompyuter sinfi

Mashg'ulotning borishi:

a) Auditoriya va talabalarni darsga tayyorligini aniqlash

b) Yo'qlama qilish.

c) Talabalardan o'tilgan mavzularni takrorlash

d) Yangi mavzuning bayoni: kompyuter va ko'rgazmali qurollar yordamida bayon etiladi (Aqliy hujum metodi)

e) Dars yakuni: Amaliy mashg'ulot bo'yicha har bir talabaga individual topshiriqlar beriladi.

Mavzuni qisqacha bayoni

1. Nyuton - Kotes formulalari:

$$J_h^{NK}(f) = J(L_n(x)) = \int_a^b \sum_{i=0}^{n} f(x_i) l_i(x) = \sum_{i=0}^{n} f(x_i) p_i, l_i(x) = \prod_{j=0, j\neq i}^{n} (x - x_j)/(x_i - x_j), p_i = \int_a^b l_i(x)dx,$$

$$R_h^{NK}(f) = J(f) - J_h^{NK}(f) = \frac{f^{(n+1)}(\xi)}{(n+1)!}\int_a^b (x-x_0)...(x-x_n)dx$$

Usulning chizmasi quyida rasm 11.1-rasmda ko'rsatilgan.

2.Simpson formulasi:

$$J_h^S(f) = \frac{h}{3}[f(x_0) + 4\sum_{i=0}^{m-1} f(x_{2i+1}) + 2\sum_{i=1}^{m-1} f(x_{2i})] + f(x_{2m})], R_h^S(f) = -\frac{f^{(4)}(\xi)(b-a)}{180}h^4$$

Usulning chizmasi quyida 11.2-rasmda ko'rsatilgan.

Topshiriqni Mathcadda bajarish.

1.Nyuton-Kotes (NK) formulalari. NK formulalari birinchi marta amaliyotga kiritilmoqda. Ilgari, bu formulalar qo'lda yoki algoritmik tillarda qiyinligi sababli hech qachon amaliy darslarda ishlatilmagan, faqatgina nazariy darslarda shaffofligi, ravshanligi tufayli ishlatilgan. Mapleda interaktiv oynasida shunday imkoniyat mavjud.

$f(x) := \sin(x)$ $a := 0$ $b := \pi$ $n := 20$ *// funksiya, oraliq, bo'linishlar soni*

$J := \int_a^b f(x)dx$ $J = 2$ *// hisoblanadigan integral*

$h := (b-a)$ $i := 0..n$ $vx_i := a + i * h$ *// qadam, tugun nuqtalar*

$l(i,x) := \prod_{j=0}^{n} if(i = j, 1, \frac{x - vx_j}{vx_i - vx_j})$ $p_i := \int_a^b l(i,x)dx$ *// Lagranj fundamental ko'phadlari*

$$JNK(n) := \sum_{i=0}^{n} f(vx_i)p_i, \quad JNK(n) = 2 \qquad \text{// } \textit{NK formulasi va qiymat}$$

Topilgan natija integralning aniq qiymat bilan birxil.

2. Simpson formulasi:

$$f(x) := \sin(x) \; a := 0 \; b := \pi \; n := 20 \qquad \text{// } \textit{funksiya, oraliq, bo'linishlar soni}$$

$$J := \int_a^b f(x)dx \quad J = 2 \qquad \text{// } \textit{hisoblanadigan integral}$$

$$h := (b-a) \; i := 0..n \; vx_i := a + i * h \qquad \text{// } \textit{qadam, tugun nuqtalar}$$

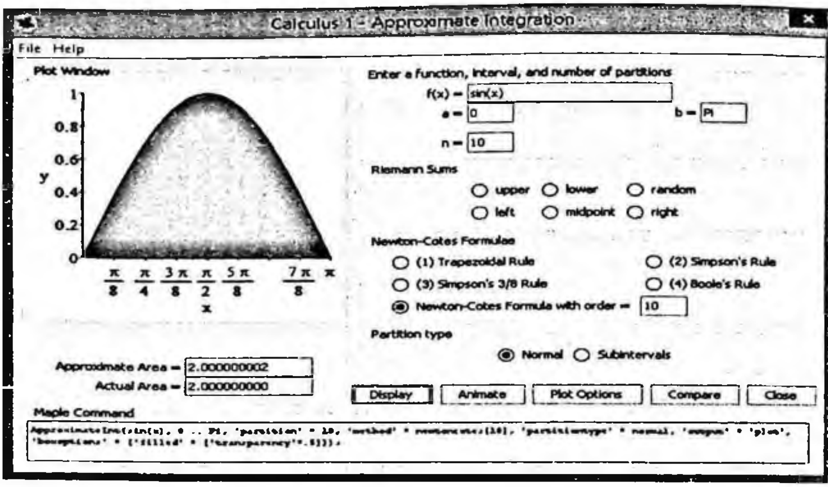
$$J_h^S(f) = \frac{h}{3}\{f(a) + 4\sum_{i=1}^{m} f(x_{2i-1}) + 2\sum_{i=1}^{m-1} f(x_{2i})] + f(b)\} \qquad \text{// } \textit{Simpson formulasi, qiymati}$$

$$JS(10) = 2.0000067844 \qquad \text{// } \textit{Integralning taqribiy qiymati}$$

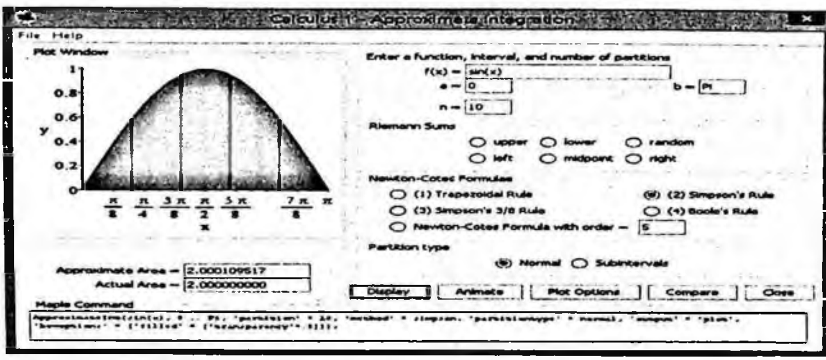
Izoh. Barcha algoritmlarni birlashtiring. (bu juda oson, chunki algoritmlarda formulalar satrgina farqli xolos).

Topshiriqlarni Mapleda bajarish.

Maple da integrallarni $\mathrm{int}(f(x), x = a..b)$ komandalar bilan bevosita hisoblash mumkin. Lekin, Maple da integrallarni interaktiv usulda ham hisoblash mumkin. Buning uchun Tools>Tutors>Calculus>Single-Variable>Approximate Integration komandasini beramiz: Bu yerda integrallash uchun funksiya kiritish, chegaralarni berish, kesmadagi oraliqlar sonini berish, Riman yig'indisini hisoblash uchun tugun nuqtalar tanlash (6 xil usul mavjud), taqribiy integrallash usullarini tanlash (5 xil usul), integralning aniq va taqribiy qiymatlarini ko'rish va animasiyani ko'rish imkoniyatlari mavjud.



11.1-rasm



11.2-rasm

Interaktiv oynada aniqlikning kamayish tartibida Nyuton-Kotes, Simpson, to'g'rito'rtburchaklar, trapesiyalar formulalari bilan hisoblashlar keltirilgan.

Individual topshiriqlar

$y=f(x)$ funksiya $[a,b]$ kesmada berilgan. To'r $x_i=a+ih, i=0..n, n=10, h=(b-a)/n$ tuzilib funksiya Nyuton-Kotes, trapesiya, to'g'ri to'rtburchaklar, Simpson formulalari bilan taqribiy integrallansin. Xatoliklar solishtirilsin. Nuqtalar soni o'zgartirilib xatolikning kamayishi kuzatilsin:

№	Funksiya	javoblar
1	$f(x)=x^4(1+x^2)^{-1},[1,2]$	
2	$f(x)=x^2e^{-2x},[0;1.6]$	
3	$f(x)=x^{-0.5}\ln(x),[1,3]$	
4	$f(x)=x\sin 3(x),[0,1]$	
5	$f(x)=\sqrt{1+x}\lg(1+x),[0.1,1.1]$	
6	$f(x)=x^2\ln(x),[1,2]$	
7	$f(x)=x^2(x^2+1)^{-1},[1,4]$	
8	$f(x)=x\cos(2x),[0,1]$	
9	$f(x)=x^4\ln(x),[1,2]$	
10	$f(x)=\sqrt{x}\ln(x),[1,4]$	
11	$f(x)=x^4(1+x^2)^{-1},[1,2]$	
12	$f(x)=x^3\sqrt{1-x^2},[1,2]$	
13	$f(x)=\sqrt{x}/(1+x),[1,4]$	
14	$f(x)=e^{-x},[1,4]$	
15	$f(x)=xarctg(x),[0,1]$	
16	$f(x)=x\arccos(x),[-0.5,0.5]$	
17	$f(x)=x\arcsin(x),[0,0.9]$	
18	$f(x)=(x+x^3)^{-1},[1,2.2]$	
19	$f(x)=x3^{-x},[0,1.5]$	
20	$f(x)=x^2e^{-x},[0,1]$	
21	$f(x)=x^3/(1+x^2),[0,2]$	
22	$f(x)=(x+x^2)^{-1},[1,3]$	
23	$f(x)=\sqrt{1+x^2},[0,2]$	
24	$f(x)=x^2\sin(x),[0,1]$	
25	$f(x)=x\sin(x),[0,1.6]$	

26	$f(x)=x^3/\sqrt{1+x^2},[-0.4,0.8]$	
27	$f(x)=x^2\cos(x),[0,1]$	
28	$f(x)=x2^{-x},[0,2]$	
29	$f(x)=e^x\sin(x),[0,1.2]$	
30	$f(x)=x^2\,\mathrm{arc}tg(x),[0,1]$	

Nazariy savollar va topshiriqlar.

1. $J(f)=\int_a^b f(x)dx$ nima uchun taqribiy hisoblanadi
2. Taqribiy integrallash formulasi nima?
3. Kvadratura formulasi nima?
4. Kvadratura formulasining koeffisientlari nima?
5. Nyuton-Kotes kvadratura formulasining tugun nuqtalari nima?
6. Simpson kvadratura formulasining qoldiq hadlari nima?
7. Kvadratura formulasining qoldiq hadlari nima?

Amaliy mashg'ulot №12

Mavzu: Oddiy differensial tenglamalarni sonli yechish usullari

Mashg'ulotning maqsadi: Talabalarda oddiy differensial tenglamalarni sonli yechish usullari ko'nikmasini shakllantirish

Mashg'ulotning jihozi:

a) Ko'rgazmali material

b) Kompyuterning dasturiy ta'minoti

c) Matematik tizimlar

Mashg'ulot texnik vositalari: Pentium IV kompyuterlari (qo'shimcha qurilmalari) bilan jihozlangan kompyuter sinfi

Mashg'ulotning borishi:

a) Auditoriya va talabalarni darsga tayyorligini aniqlash

b) Yo'qlama qilish.

c) Talabalardan o'tilgan mavzularni takrorlash

d) Yangi mavzuning bayoni: kompyuter va ko'rgazmali qurollar yordamida bayon etiladi (Aqliy hujum metodi)

e) Dars yakuni: Amaliy mashg'ulot bo'yicha har bir talabaga individual topshiriqlar beriladi.

Mavzuni qisqacha bayoni

1. ODT uchun Koshi masalasini taqribiy yechish

ODT uchun Koshi masalasini yechish uchun

$x \in [a,b]$ kesmada ushbu to'rni: $\Delta_n : a = x_0 < x_1 < ... < x_n = b, x_{i+1} - x_i = h$ qaraymiz. F.q. $y(x_i)$-yechimning x_i nuqtadagi qiymati bo'lsin. Uning nuqtadagi taqribiy qiymatini $y_i \approx y(x_i)$ deb belgilaymiz, nuqtadagi xatolik $r_i = y(x_i) - y_i, i = 0..n$, ga teng. Bu masalani yechish uchun 4 ta bir qadamli Runge-Kutta usullari: Eyler (E1), takomillashgan Eyler (TE2), prognoz-korreksiya (PK2), 3-tartibli Simpson tipidagi usul, 4-tartibli Runge-Kutta (RK4) usullari:

1) $y_{i+1} := y_i + hf(x_i, y_i), r_i = O(h),$ (E)

2) $y_{i+1} := y_i + hf(x_i + \frac{h}{2}, y_i + \frac{h}{2} f(x_i, y_i)), r_i = O(h^2)$, (TE)

3) $y_{i+1} := y_i + h(f(x_i, y_i) + f(x_{i+1}, y_{i+1}))/2, \ r_i = O(h^2),$ (PK)

4) $I_{i+1} = I_i + (h/6)[f(x_i, I_i) + 4f(x_i + h/2, I_i + 0.5hf(x_i, I_i)) + f(x_i + h, I_i + hf(x_i, I_i))]$ (I)

5) $y_{i+1} := y_i + \frac{1}{6}(k1_i + 2k2_i + 2k3_i + k4_i), \ r_i = O(h^4),$ (RK)

$k1_i = hf(x_i, y_i) \ k2_i = hf(x_i + h/2, y_i + k1_i/2)$

$k3_i = hf(x_i + h/2, y_i + k2_i/2) \ k4_i = hf(x_i + h, y_i + k3_i)$

Mathcadda ushbu komandalarni yozamiz:

Runge-Kutta usullari gruppasi

$a := 1 \ b := 2 \ n := 10 \ h := (b-a)/n \ i := 0..n \ x_i := a + i * h$

$f(x,y) := x + y/x \ ua(x) := x + x * x \ ua_i := ua(x_i)$

$u_0 := 2 \ E_0 := u_0 \ T_0 := u_0 \ P_0 := u_0 \ R_0 := u_0 \ I_0 := u_0$

$i := 0..n-1$

$E_{i+1} := E_i + h * f(x_i, E_i)$

$T_{i+1} := T_i + h * f(x_i + h/2, T_i + h * f(x_i, T_i)/2)$

$P_{i+1} := P_i + h * (f(x_i, P_i) + f(x_{i+1}, P_i + h * f(x_i, P_i))/2$

$k1(x,y) := h * f(x,y) \ k2(x,y) := h * f(x + h/2, y + k1(x,y)/2)$

$k3(x,y):=h*f(x+h/2,y+k2(x,y)/2)\quad k4(x,y):=h*f(x+h,y+k3(x,y))$

$R_{i+1}:=R_i+(k1(x_i,R_i)+2*k2(x_i,R_i)+2*k3(x_i,R_i)+k4(x_i,R_i))/6$

$I_{i+1}:=I_i+(1/6)*[h*f(x_i,I_i)+4*[h*f(x_i+h/2,I_i+(h/2)*f(x_i,I_i)]+h*f(x_i+h,I_i+h*f(x_i,I_i))]$

$j:=0..n\quad C_{0,j}:=ua_j\quad C_{1,j}:=E_j\quad C_{2,j}:=T_j\quad C_{3,j}:=P_j\quad C_{4,j}:=R_j\quad C_{5,j}:=I_j$

12.1-jadval

C =

	0	1	2	3	4	5	6	7	8	9	10
0	2	2.31	2.64	2.99	3.36	3.75	4.16	4.59	5.04	5.51	6
1	2	2.3	2.6191	2.9573	3.3148	3.6916	4.0877	4.5032	4.9381	5.3924	5.8662
2	2	2.3098	2.6395	2.9893	3.359	3.7488	4.1586	4.5883	5.0381	5.5078	5.9976
3	2	2.2959	2.6109	2.9451	3.2986	3.6714	4.0635	4.475	4.9059	5.3563	5.8261
4	2	2.31	2.6291	2.9673	3.3248	3.7016	4.0977	4.5132	4.9481	5.4024	5.8762
5	2	2.3074	2.6347	2.9818	3.3488	3.7356	4.1424	4.5691	5.0156	5.4821	5.9684

Natijalar 5 ta usulda aniq yechim qiymatlari bilan mos.

Izoh. I, R formulalar avtonom yozuvga ega, ular boshqa usullardan bog'liq emas.

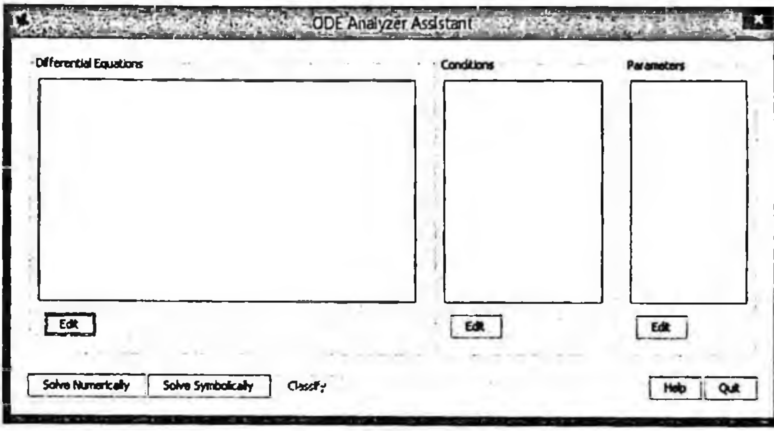
(8) formulani quyidagicha ham yozish mumkin:

$k(x,y):=hf(x,y)$

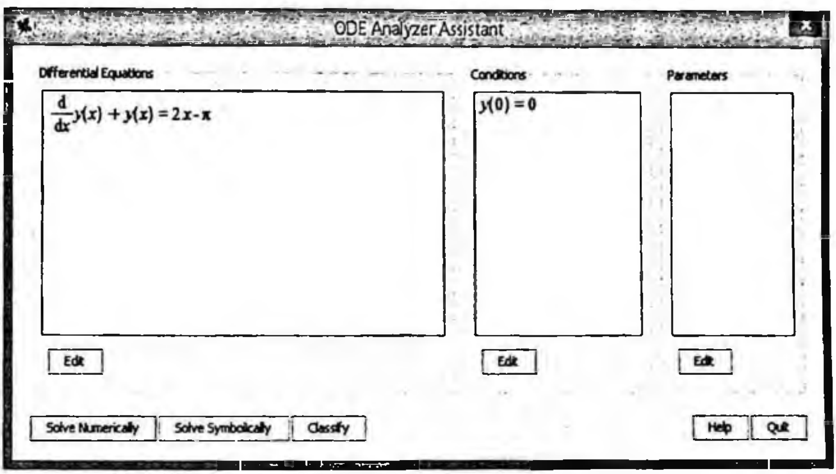
$I_{i+1}:=I_i+(1/6)*[k(x_i,I_i)+4*k(x_i+h/2,I_i+0.5*k(x_i,I_i))+k(x_i+h,I_i+k(x_i,I_i))]$

Topshiriqni Maple tizimida bajarish.

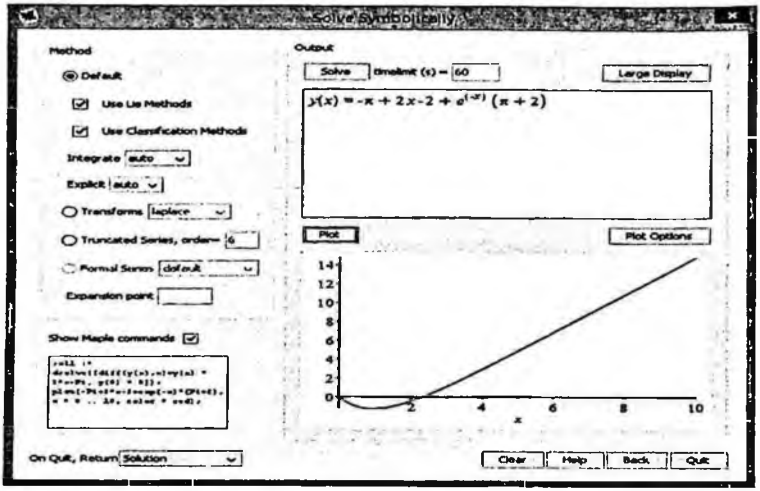
Tools>Assistants>ODE Analizer komandasi yordamida ODT uchun Koshi yoki chegara masalanini interaktiv usulda analitik yoki sonli yechish mumkin. ODE Analizer Assistant muloqot oynasida ochiladi. Bu yerda avval, Differential Equation oynasiga tegishli Edit tugmasi bosilib, differensial tenglama, so'ng Conditions oynasiga tegishli Edit tugmasi bosilib boshlang'ich shart kiritiladi. Endi, tenglamani 2 xil yo'l bilan yechish mumkin: Solve Symbolically (simvolli yechish) va Solve Numerically (taqribiy yechish). Avval simvolli echamiz. Bu yerda klassik yechim va uning grafigini chiqarish mumkin. Tenglamani taqribiy yechish oynasi usullarga juda boy. Avvalo, 9 ta usulni Parametrs bo'limidan tanlash mumkin. Bu yerda oddiy differensial tenglamadan tortib, qattiq oddiy differensial tenglamalar sistemasi va ular uchun chegara masalalarni yechish imkoniyatini berruvchi interaktiv oynalar mavjud. Maxsus oynada bu ishlarga mos keluvchi Maple tizimi komandalarini ham ko'rish mumkin. Taqribiy yechimni biror nuqtada qiymatini chiqarish mumkin. Grafigini tasvirlash mumkin. So'ng, yana 9 ta usulni Fixed step methods (qadami fiksirlangan usullar: Eyler, Runge-Kutta, Adams usullari) bo'limidan tanlash mumkin. Ularni biz ekranda ochib qo'ydik.



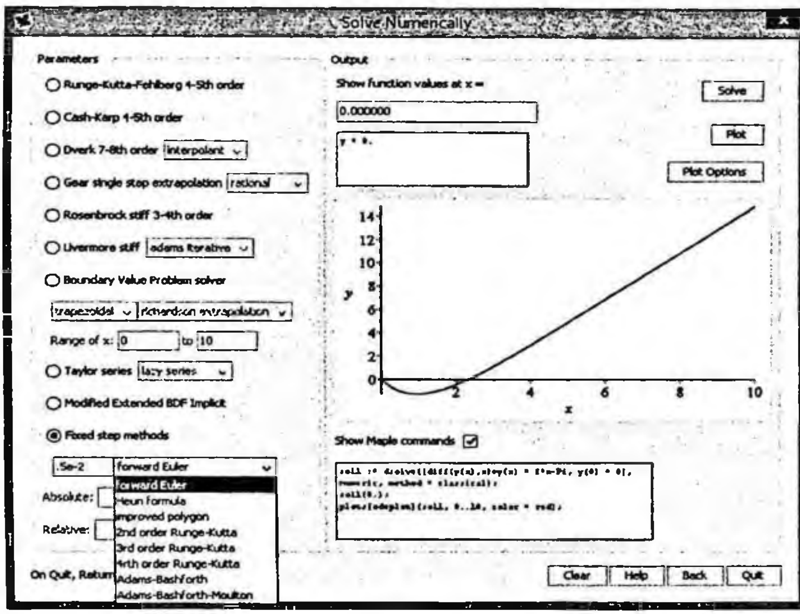
12.1.1-rasm



12.1.2-rasm



12.1.3-rasm



12.1.4-rasm

Individual topshiriqlar.

ODT uchun Koshi masalasining taqribiy yechimi Eyler, Runge-Kutta usullari bilan Mathcad, Maple matematik tizimlarida hisoblansin. Natijalarning o'xshashligiga yerishilsin. Grafiklar chizilsin.

12.2.-jadval

	ODT	[a,b]	BSH	h	Aniq yechim
1	$y' - xy = -x^2 \sin x$	[0, 0.5]	$y(0)=1$	0.1	$\cos x + x\sin x$
2	$(1+x)y' - y = 2 - 2\ln(1+x)$	[0, 0.5]	$y(0)=1$	0.05	$1 + x + 2\ln(1+x)$
3	$y' - y = e^{-x}(\cos x - \sin x)$	[0, 0.5]	$y(0)=1$	0.05	$e^{-x}(\cos x + \sin x)$
4	$y' + 2y = e^{-2x}$	[0,1]	$y(0)=1$	0.1	$(1+x)e^{-2x}$
5	$y' - 2y = -4\cos 2x$	[0, 0.2]	$y(0)=1$	0.02	$\cos 2x - \sin 2x$
6	$y' - y = 2e^{-x}$	[0,2]	$y(0)=2$	0.2	$e^x + e^{-x}$
7	$y' - 5y = -2e^{3x}$	[0, 0.2]	$y(0)=2$	0.02	$e^{5x} + e^{3x}$
8	$y' + 2y = 4\cos 2x$	[0,1]	$y(0)=1$	0.1	$\cos 2x + \sin 2x$
9	$y' + y = 6e^{3x}$	[0,1]	$y(0)=2$	0.1	$e^{3x} + e^{-3x}$
10	$y' - y = e^x$	[0,1]	$y(0)=1$	0.1	$e^x(1+x)$
11	$y' + y = 5e^{2x}$	[0,1]	$y(0)=0$	0.1	$e^{2x} - e^{-3x}$
12	$y' + (3/4)y = (5/4)e^{x/2}$	[0,1]	$y(0)=2$	0.1	$e^{x/2} + e^{-3x/4}$
13	$xy' - y = x^2 - x - 1$	[1,2]	$y(0)=0$	0.1	$x^2 + x$
14	$y' + 3y = 0$	[0,0.5]	$y(0)=1$	0.05	$e^{-3x}(\cos x + \sin x)$
15	$y' + 2y = 2.5e^{3x} + 1.5e^x$	[0,1]	$y(0)=0$	0.02	$-e^{2x} + 0.5e^{3x} + 0.5e^x$
16	$y' - y = 0.1e^{2x} - 0.5x^2 - 2x - 1$	[0,1.5]	$y(0)=5.1$	0.1	$e^x + 0.1e^{2x} + 0.5x^2 + 3x + 4$
17	$y' + y = 2\cos x + 1 + e^x$	[1,1.5]	$y(0)=2.5$	0.1	$\cos x + \sin x + 1 + e^x/2$
18	$x^2y - xy = -2$	[1,1.5]	$y(0)=4$	0.05	$2x + 1/x + x^2$
19	$xy' + y = 5\ln x - 1$	[1,1.5]	$y(1)=5$	0.05	$5 - \ln x$
20	$y' - y = e^x - x^2e^x/3$	[0,0.5]	$y(0)=1$	0.05	$(1+x)e^x + x^3e^x/6$
21	$y - 2y = 2e^{3x}$	[0,1]	$y(0)=2$	0.1	$e^x + e^{2x}$
22	$2xy' - y = 4x^{-2}$	[1,2]	$y(0)=2$	0.1	$3\sqrt{x} - x^{-2}$

23	$2\sqrt{x}y'+y=2\cos\sqrt{x}$	[1,2]	$y(0)=1$	0.1	$\sin\sqrt{x}+\cos\sqrt{x}$
24	$10y'-3y=20x-0.3x^3-1$	[1,2]	$y(0)=1/3$	0.1	$x^2+0.1x^3+1/3$
25	$y'+y=2e^x$	[0,1]	$y(0)=2$	0.1	e^x+e^{-x}
26	$y'+2y=4\cos 2x$	[0,0.5]	$y(0)=1$	0.05	$\sin 2x+\cos 2x$
27	$y'+0.5y=1.5e^x$	[0,1]	$y(0)=1$	0.1	$e^x+e^{-x/2}$
28	$y'-y=xe^{-2x}-2(1+x)$	[0,0.5]	$y(0)=1$	0.05	$(1+x)e^{-2x}$
29	$y'-y=\cos(x)-\sin(x)$	[0,1]	$y(0)=1$	0.01	$\sin(x)$
30	$y'-y=-\sin(x)-\cos(x)$	[0,1]	$y(0)=-1$	0.01	$\cos(x)$

1. $y'=f(x,y)=\sin(ax)+y^2/10, y(0)=0,[a,b]=[0,1]$ .2. $y'=f(x,y)=\cos(ax)+y^2/10, y(0)=0,[a,b]=[0,1]$

Nazariy savollar va topshiriqlar.

1. ODT uchun Koshi masalasi nima?

2. ODT uchun Koshi masalasida taqribiy analitik usullar nima?

3.ODT uchun sonli usullar nima?

4. Takomillashgan Eyler usuli, aniqligi, algoritmi?

5. Eylerning prognoz-korreksiya usuli, aniqligi, algoritmi?

6.Runge-Kutta usullarining aniqligi, algoritmi?

7.Ko'p qadamli Adams usullarining aniqligi, algoritmi?

2. ODT uchun chegara masalalarni taqribiy yechish

Qisqacha nazariy malumot.

ODT uchun chegara masala quyidagicha qo'yiladi:

$Lu \equiv u''+p(x)u'+q(x)u=f(x),\ a\le x\le b,$ (differensial tenglama-DT), (1)

$l_0u\equiv\alpha_0u(a)+\alpha_1u'(a)=\gamma_0,\ l_1u\equiv\beta_0u(b)+\beta_1u'(b)=\gamma_1$ (chegara shartlar-CHSH). (2)

Quyida ushbu chegara masala echiladi:

$p(x)=x^2\ q(x)=-x\ \ f(x)=6/x^4-3/x\ \ ua(x):=1/x^2\ \ a=1\ \ b=2$

DT va CHSH qanoatlantiruvchi $u=u(x)\in C^2[a,b]$ funksiyani topish kerak. Proeksion (kollokasiya, Galyorkin, Rits, eng kichik kvadratlar...) usullarda taqribiy yechim $u_n(x)\approx u(x)$ quyidagicha izlanadi:

$$u_n(x)=\varphi_0(x)+\sum_{j=1}^{n}c_j\varphi_j(x),\ c_j\text{-?}.$$

Bu yerda $\{\varphi_i(x)\}$ lar bazis funksiyalar, ular maxsus usul bilan tanlanadi.

Kollokasiya usulida koeffisentlar quyidagicha topiladi:

$$Lu_n(x_i)=f(x_i), i=1..n \Leftrightarrow \sum_{j=1}^{n}c_jL_j\varphi_j(x_i)=f(x_i)-L\varphi_0(x_i), i=1..n; x_i\in(a,b).$$

Eng kichik kvadratlar usulida koeffisentlar quyidagicha topiladi:

$$Lu_n(x)-f(x)\perp L\varphi_i(x)\Leftrightarrow \sum_{j=1}^{n}c_j(L\varphi_j,L\varphi_i)=(f-L\varphi_0,L\varphi_i), i=1..n \quad (f,g)=\int_b^a f(x)g(x)dx$$

Galyorkin-Rits usulida koeffisentlar quyidagicha topiladi:

$$Lu_n(x)-f(x)\perp\varphi_i(x)\Leftrightarrow\sum_{j=1}^{n}c_j(L\varphi_j,\varphi_i)=(f-L\varphi_0,\varphi_i),\ (f,g)=\int_b^a f(x)g(x)dx,\ i=1,..n$$

Bazis funsiyalarni tanlash:

1-tur chegara shartlar: u(a)=A, u(b)=B, $\varphi_0(x)=A+(B-A)(x-a)/(b-a)$,

a) $\varphi_i(x)=(x-a)^i(b-x), i\ge 1$; b) $\varphi_i(x)=\sin(\pi i(x-a)/(b-a)),\quad i\ge 1.$

2-tur chegara shartlar: u'(a)=A, u'(b)=B,

$$\varphi_0(x)=Ax+(B-A)(x-a)^2/(2(b-a)),$$

a) $\varphi_i(x)=(x-a)^{i+1}(b-x)^2,\ i\ge 1$; b) $\varphi_i(x)=cos(i\pi(x-a)/(b-a)),\ i\ge 1$

3-tur chegara shartlar: $\alpha_0u(a)+\alpha_1u'(a)=A,\ \beta_0u(b)+\beta_1u'(b)=B$,

Aytaylik, $\varphi_0(t)=kt+d$, u holda $(\alpha_0a+\alpha_1)k+\alpha_0d=A,\ (\beta_0b+\beta_1)k+\beta_0d=B$. Kram yer qoidasiga asosan, $k=D_k/D, d=D_d/D$, bu yerda

$$D=\begin{vmatrix}\alpha_0a+\alpha_1 & \alpha_0\\ \beta_0b+\beta_1 & \beta_0\end{vmatrix},\ D_k=\begin{vmatrix}A & \alpha_0\\ B & \beta_0\end{vmatrix},\ D_d=\begin{vmatrix}\alpha_0a+\alpha_1 & A\\ \beta_0b+\beta_1 & B\end{vmatrix}.$$

$a)\varphi_i(x)=(x-a)^{i+1}(b-x)^2;\ b)\ \varphi_i(x)=(x-a)^{i+1}(\gamma_i-x),\ i\ge 1,\ \gamma_i=b+\beta_1L/[\beta_0L+\beta_1(i+1)], i\ge 1$.

Masalaning Mathcadda yechishni tashkil etish

Mathcadda ushbu komandalarni yozib chiqamiz:

$ORIGIN:=1\ p(t):=t^2\ q(t):=-t\ f(t):=(6/t^2-3/t)\ ua(t):=1/t^2\ a:=1\ b:=2\ L:=b-a$

$n:=4\ h:=L/n\ i:=1..n\ x_i=a+ih/(n+1)\ \alpha 0:=1\ \alpha 1:=0\ A:=1\ \beta 0:=3\ \beta 1:=1\ B:=0.5$

$$MD:=\begin{bmatrix}\alpha 0a+\alpha 1 & \alpha 0\\ \beta 0b+\beta 1 & \beta 0\end{bmatrix}\ MDk:=\begin{bmatrix}A & \alpha 0\\ B & \beta 0\end{bmatrix}\ MDd:=\begin{bmatrix}\alpha 0a+\alpha 1 & A\\ \beta 0b+\beta 1 & B\end{bmatrix}$$

$D:=|MD|\ Dk:=|MDk|\ Dd:=|Mdd|\ D=-4\ Dk=2.5\ Dd=-0.625\ k:=Dk/D\ d:=Md/D$

$funksiya\ \varphi 0(t):=kt+d\rightarrow -0.625t+1.625\ \varphi 0(t):=-0.625t+1.625$

$$funksiya\ \varphi(j,t):j:=1..n\ \ \gamma_j:=b+\frac{\beta 1L}{\beta 0L+\beta 1(j+1)}\ \ \ \varphi(j,t):=(t-a)^{j+1}(\gamma_j-t)$$

$$\psi(j,t):=\frac{d^2\varphi(j,t)}{dt^2}+p(t)\frac{d\varphi(j,t)}{dt}+q(t)\varphi(j,t)\qquad \chi(t):=f(t)-\frac{d^2\varphi 0(t)}{dt^2}+p(t)\frac{d\varphi 0(t)}{dt}+q(t)\varphi 0(t)$$

$i:=1..n\ \ aK_{i,j}:=\psi(j,x_i)\ bK_i:=\chi(x_i)\ |aK|=9.515*10^{-4}\ cK:=aK^{-1}bK$

$$aG_{i,j}:=\int_a^b\psi(j,t)\varphi(i,t)dt\ \ bG_i:=\int_a^b\chi(t)\varphi(i,t)dt\ \ |aG|=9.515*10^{-10}\ \ cG:=aG^{-1}bG$$

$$aE_{i,j}:=\int_a^b\psi(j,t)\psi(i,t)dt\ \ bE_i:=\int_a^b\chi(t)\psi(i,t)dt\ \ |aE|=0.012\ \ cE:=aE^{-1}bE$$

$$uK(t):=\varphi 0(t)+\sum_{i=1}^{n}cK_i\varphi(i,t)\ \ uK_i:=uK(x_i)\ \ U_{1,j}:=uK_j$$

$$uG(t):=\varphi 0(t)+\sum_{i=1}^{n}cG_i\varphi(i,t)\ \ uG_i:=uG(x_i)\ \ U_{2,j}:=uG_j$$

$$uE(t):=\varphi 0(t)+\sum_{i=1}^{n}cE_i\varphi(i,t)\ \ uE_i:=uE(x_i)\ \ U_{3,j}:=uE_j\ \ ua_j:=ua(x_j)\ U_{4,j}:=ua_j$$

$$U=\begin{bmatrix}0.974 & 0.957 & 0.947 & 0.942\\ 0.933 & 0.822 & 0.696 & 0.574\\ 0.972 & 0.95 & 0.932 & 0.916\\ 0.907 & 0.826 & 0.756 & 0.694\end{bmatrix}.\quad \left(U=\begin{bmatrix}0.963 & 0.945 & 0.93 & 0.92\\ 0.945 & 0.87 & 0.79 & 0.716\\ 0.963 & 0.927 & 0.89 & 0.853\\ 0.907 & 0.826 & 0.756 & 0.694\end{bmatrix}\right)$$

Izoh. Qavs ichidagi yechim quyidagi holga mos: $\alpha 0=\beta 0=1, \alpha 1=\beta 1=0, A=B=1$

ODT uchun chekli ayirmali sxemani Mathcad da yechish.

ODT uchun chekli ayirmali sxema (CHAS) quyidagi ko'rinishga ega:

$$L_h u_h=\left\{\frac{u_{i-1}-2u_i+u_{i+1}}{h^2}+p_i\frac{u_{i+1}-u_{i-1}}{2h}+q_iu_i=f_i,\ i=0,...,n-1,\right\}=f_h, \tag{3}$$

$$l_h u_h = \{l_{0h}u_h, l_{1h}u_h\} = \{\alpha_0 u_0 + \alpha_1 \frac{u_1 - u_0}{h}, \beta_0 u_n + \beta_1 \frac{u_n - u_{n-1}}{h}\} = g_h = \{\gamma_0, \gamma_1\}. \qquad (4)$$

Uni quyidagi ko'rinishga keltiramiz:

$$\begin{cases} (\alpha_0 h - \alpha_1)u_0 + \alpha_1 u_1 = \gamma_0 h, \\ (1 - p_i h/2)u_{i-1} + (q_i h^2 - 2)u_i + (1 + p_i h/2)u_{i+1} = f_i h^2, \qquad i = 1,\dots,n-1 \\ -\beta_1 u_{n-1} + (\beta_0 h + \beta_1)u_n = \gamma_1 h \end{cases}$$

Quyidagicha belgilashlar kiritib

$$b_0 = \alpha_0 h - \alpha_1, c_0 = \alpha_1, d_0 = \gamma_0 h, \ a_i = 1 - p_i h/2, b_i = q_i h^2 - 2, c_i = 1 + p_i h/2, d_i = f_i h^2,$$

$$i=1,..,n-1, \ a_n = -\beta_1, b_n = \beta_0 h + \beta_1 \ d_n = \gamma_1 h,$$

standart uch dionalli ko'rinishga keltiramiz:

$$\{b_0 u_0 + c_0 u_1 = d_0, \ a_i u_{i-1} + b_i u_i + c_i u_{i+1} = d_i, i = 1,\dots,n-1, a_n u_{n-1} + b_n u_n = d_n.$$

$$Au = d, \ A = \begin{bmatrix} b_0 & c_0 & 0 & . & 0 & 0 & 0 \\ a_1 & b_1 & c_1 & . & 0 & 0 & 0 \\ 0 & a_2 & b_2 & . & 0 & 0 & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & b_{n-2} & c_{n-2} & 0 \\ 0 & 0 & 0 & . & a_{n-1} & b_{n-1} & c_{n-1} \\ 0 & 0 & 0 & . & 0 & a_n & b_n \end{bmatrix}, \ u = \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ . \\ u_{n-2} \\ u_{n-1} \\ u_n \end{bmatrix}, \ d = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ . \\ d_{n-2} \\ d_{n-1} \\ d_n \end{bmatrix} \qquad (5)$$

Uch diognalli sistema progonka (haydash) usuli bilan echiladi. Haydash usulida uchdiognalli sistema yechimi quyidagicha izlanadi: $u_i = v_{i+1}u_{i+1} + w_{i+1}$, bu yerda progonka koeffisientlari $\{v_i \ w_i$ va nomalumlar $\{u_i\}$ ushbu formulalar yordamida topiladi:

$\{v_{i+1} = -c_i/[a_i v_i + b_i], w_{i+1} = [d_i - a_i w_i]/[a_i v_i + b_i], \ i = 1,\dots n-1, v_0 = w_0 = 0,$ // *o'ng yurish*

$u_n = [d_n - a_n w_n]/[a_n v_n + b_n], u_i = v_{i+1}u_{i+1} + w_{i+1}, i = n-1, n-2,\dots,0.$ // *chap yurish*

Masalani Mathcadda yechish.

ODT sifatida ushbu tenglamani olamiz: $u''(t) + p(t)u'(t) + q(t)u(t) = f(t)$.

$p(t) := t^2, \ q(t) := -t, \ f(t) := 6/t^4 - 3/t, \ a = 1, \ b = 2, \ \overline{u}a(t) = 1/t^2$ // koeffisientlar

n:=10 i:=0..n t_0:=1 t_n:=2 h:=(t_n-t_0)/n $t_i := t_0 + ih$ // *kesma, parametrlar*

$\alpha 0 := 1 \ \alpha 1 := 0 \ \ \gamma 0 := 1$ // *CHSH koeffisientlari*

$\beta 0:=3$ $\beta 1:=1$ $\gamma 1:=0.5$ *// CHSH koeffisientlari*

$b_0=\alpha 0h-\alpha 1, c0=\alpha 1, d_0=\gamma 0h,$ *//0-tenglama koef.lari*

$a_n=-\beta 1. b_n=\beta 0h+\beta 1$ $d_n:=\gamma 1h$ *//n-tenglama koeffisientlari*

i=1,..,n-1, $a_i=1-p_ih/2, b_i=q_ih^2-2, c_i=1+p_ih/2, d_i=f_ih^2$ *//i-tenglama koef.sientlari*

$ut(x):=1/x^2$ $\quad ut_i:=ut(x_i)$ *// aniq yechim va uning qiymatlari*

$m_{0,0}:=\alpha 0h-\alpha 1$ $m_{0,1}:=\alpha 1$ $d_0:=\gamma 0h$ *// Mathcadda 0-tenglama koeffisientlari*

$i:=1..n-1$ $m_{i,i-1}:=1-p(x_i)\frac{h}{2}$ $m_{i,i}:=b_i$ $m_{i,i+1}:=1+p(x_i)\frac{h}{2}$ $d_i:=f(x_i)h_i^2$ // i-tenglama koef.

$m_{n,n-1}:=\beta 1 m_{n,n}:=\beta 0h+\beta 1$ $d_n:=\gamma 1h$ *// Mathcadda n-tenglama koeffisientlari*

$u:=m^{-1}d$ *// CHAS ni yechish va ekranga chiqarish taqribiy va aniq yechimni chiqarish*

12.2.1-jadval

$u^T=$

0	1	2	3	4	5	6	7	8	9	10
1	0.8276	0.6963	0.5941	0.5131	0.4478	0.3944	0.3502	0.3132	0.282	0.2554

12.2.2-jadval

$ua^T=$

0	1	2	3	4	5	6	7	8	9	10
1	0.8264	0.6944	0.5917	0.5102	0.4444	0.3906	0.346	0.3086	0.277	0.25

Aniq va taqribiy yechim o'nli verguldan so'ng ikki xonagacha bir xil.

Nazorat uchun ayirmali sxema matrisasi, o'ng tomonni chiqaramiz:

12.2.3-jadval

m =

	0	1	2	3	4	5	6	7	8	9	10
0	0.1	0	0	0	0	0	0	0	0	0	0
1	0.939	2.011	1.061	0	0	0	0	0	0	0	0
2	0	0.928	2.012	1.072	0	0	0	0	0	0	0
3	0	0	0.915	2.013	1.085	0	0	0	0	0	0
4	0	0	0	0.902	2.014	1.098	0	0	0	0	0
5	0	0	0	0	0.888	2.015	1.113	0	0	0	0
6	0	0	0	0	0	0.872	2.016	1.128	0	0	0
7	0	0	0	0	0	0	0.855	2.017	1.145	0	0
8	0	0	0	0	0	0	0	0.838	2.018	1.162	0
9	0	0	0	0	0	0	0	0	0.82	2.019	1.18
10	0	0	0	0	0	0	0	0	0	-1	1.3

d =

	0
0	0.1
1	0.014
2	0.004
3	-0.002
4	-0.006
5	-0.008
6	-0.01
7	-0.01
8	-0.011
9	-0.011
10	0.05

Jadvaldan ko'rinadiki taqribiy yechimlar 0.01 xatolik bilan bir xil.

Chegara masalani Koshi masalasiga keltirish usullari

Masalaning qo'yilishi: $y''(x) = f(x, y, y'),\ y(a) = A,\ y(b) = B.$ (6)

1. Otish usuli.

$$y''(x) = f(x, y, y'), y(a) = A, y'(a) = C : y = y(b, C), g(C) = y(b) - y(b, C) = 0. \quad (7)$$

2. Reduksiya usuli.

$$Ly = f, l_a y = A, l_b y = B, y = Cu(x) + v(x) \Leftrightarrow \begin{cases} 1) Lu = 0, u(a) = \alpha_1, u'(a) = -\alpha_0, & (8) \\ 2) Lv = f, v(a) = A/\alpha_0, v'(a) = 0, & (9) \\ (v(a) = 0, v'(a) = A/\alpha_1), C = (B - l_b[v])/l_b[u]. \end{cases}$$

3. Differensial progonka usuli.

(3), (4) masala yechimi orasida usbu bog'lanish bor deylik:

$$y' = d(x)y + g(x), d, g = ?. \quad (10)$$

Nomalum funcsiyalar $d, g = ?$ ni topaylik. Keyin asosiy nomalum $y(x)$ni topamiz. Ravshanki, $y'' = d'y + dy' + g'$. Uni tenglamaga qo'yamiz:

$$d'y + dy' + g' + py' + qy = f \Rightarrow (d + p)y' + (d' + q)y + g' = f \Rightarrow y' = (-(d' + q)y + (f - g'))/(d + p)$$

Oxirgi tenglikni yechim ro'rinishlarini solishtirib topamiz:

$$d = -(d' + q)/(d + p), g = (f - g')/(d + p).$$

Demak, ushbu ODTlarni topamiz: $d' = -pd - d^2 - q,\ g' = -(d+p)g + f$. Bu ODTlar uchun boshlang'ich shartlarni topamiz:

$$y'(a) = d(a)y(a) + g(a), y'(a) = -\alpha_0 y(a)/\alpha_1 + A/\alpha_1$$

Bu shartlardan kelib chiqadi: $d(a) = -\alpha_0/\alpha_1,\ g(a) = A/\alpha_1$.

Demak, biz ODTlar uchun usbu Koshi masalalariga kelamiz:

$$d' = -pd - d^2 - q, d(a) = -\alpha_0/\alpha_1, \qquad (11)$$

$$g' = -(d+p)g + f, g(a) = A/\alpha_1 \qquad (12)$$

(11), (12) ODT lar uchun Koshi masalasi a dan b ga qarab yechiladi.

Endi $y'(x) = d(x)y(x) + g(x)$ ODT uchun boshlang'ich shart yaratamiz. Buning uchun (4) chegara shartga $y'(a) = d(a)y(a) + g(a)$ ni yoki $y'(b) = d(b)y(b) + g(b)$ ni qo'yamiz:

$$l_a[y] = A \Rightarrow \alpha_0 y(a) + \alpha_1(d(a)y(a) + g(a)) = A \Rightarrow y(a) = (A - \alpha_1 g(a))/(\alpha_0 + \alpha_1 g(a)) \Rightarrow$$
$$y(a) = (A - \alpha_1 g(a))/(\alpha_0 + \alpha_1 d(a))$$

Demak, biz quyidagi ODT uchun Koshi masalalarini hosil qilamiz:

$$y'(x) = d(x)y(x) + g(x),\ y(a) = (A - \alpha_1 g(a))/(\alpha_0 + \alpha_1 g(a)). \qquad (13)$$

$$y'(x) = d(x)y(x) + g(x),\ y(b) = (B - \beta_1 g(b))/(\beta_0 + \beta_1 g(b)) \qquad (14)$$

Bu (13) ODT uchun Koshi masalasi ham a dan b ga qarab yechiladi. (14) Koshi masalasi b dan a ga qarab yechiladi. Bunda Runge-Kutta formulalarini o'zgartirib olish kerak. (11), (12), (13) Koshi masalalarini Mathcadning standart ichki funcsiyalari rkfixed, Rkadapt, Bulstoer bilan yechamiz:

$ORIGIN := 1 \quad \alpha_0 := \quad \alpha_1 := \quad \beta_0 := \quad \beta_1 := \quad \beta_1 := \quad A := \quad B :=$

$a := 0 \quad b := 1 \quad n := 100 \quad da := \quad ga := \quad ya := \quad p(x) := \quad q(x) := \quad f(x) :=$

$$f(x,z) := \begin{bmatrix} -p(x) * z_1 - z_1^2 - q(x) \\ -(z_1 + p(x)) * z_2 + p(x) \\ z_1 * z_3 + z_2 \end{bmatrix} \quad z := \begin{bmatrix} da \\ ga \\ ya \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$Z := rkfixed(z,a,b,n,f) \quad U = Rkadapt(z,a,b,n,f) \quad V = Bulstoer(z,a,b,n,f)$

12.2.4-jadval

Z =

	1	2	3	4
1	0	0	1	1
2	0.1	-0.2	.005	1.09
3	0.2	.397	.017	.157
4	0.3	-0.59	.036	.201
5	0.4	.776	.059	.223
6	0.5	.951	.084	.224
7	0.6	.114	.108	.208
8	0.7	.262	.129	.178
9	0.8	.392	.144	138
10	0.9	-1.5	.148	.091
11	1	.584	.139	.041

U =

	1	2	3	4
1	0	0	1	1
2	0.1	-0.2	005	.09
3	0.2	397	017	157
4	0.3	.59	036	201
5	0.4	776	059	223
6	0.5	951	084	224
7	0.6	114	108	208
8	0.7	262	129	178
9	0.8	392	144	138
10	0.9	-1.5	148	091
11	1	585	139	041

V =

	1	2	3	4
1	0	0	1	1
2	0.1	-0.2	.005	1.09
3	0.2	397	.017	.157
4	0.3	0.59	036	201
5	0.4	.776	.059	.223
6	0.5	951	.084	.224
7	0.6	114	.108	.208
8	0.7	.262	.129	.178
9	0.8	392	.144	.138
10	0.9	-1.5	.148	091
11	1	.585	.139	.041

Masalani Maple matematik tizimida yechish.

Yuqorida keltirilgan interaktiv oynadan tashqari, aniq yechimni ham topish imkoniyati mavjud:

Mapleda ushbu komandalarni t yerib aniq yechimni toping.

```
restart;
ode := diff(u(x), x$2)+x^2*(diff(u(x), x))-x*u(x) = 6/x^4-3/x;
 bc1 := u(1)+0*(D(u))(1) = 1;                     // u(1) = 1
bc2 := 3*u(2)+(D(u))(2) = .5;                    // 3 u(2) + D(u)(2) = 0.5
solution := dsolve({bc1, bc2, ode});              //u(x)=1/x^2
```

Individual topshiriqlar.

ODT uchun chegara masala kolokasiya, EKK, Galyorkin, CHAS usullari bilan Mathcad, Maple matematik tizimlard echilsin. Natijalarining yaqinligiga yerishilsin. Grafiklari chizilsin.

12.2.4-jadval

	Tenglama va chegara shartlar	Yechimlar
1	$y''+xy'+y=-x\sin x,\ y(0)=1, y(\pi)=-1$	
2	$y''+xy'+y=x\cos(x),\ y(0)=0, y(\pi)=0$	
3	$y''+xy'-y=x\exp(x),\ y(0)=1, y(1)=e$	
4	$y''-xy'+y=x\sin(x),\ y(0)=1, y(\pi)=-\pi$	
5	$y''-xy'+y=-x\cos(x),\ y(0)=0, y(\pi)=0$	
6	$y''+x^2y'+y=-x^2\sin(x), y(0)=1, y(\pi)=-1$	

7	$y''+x^2y'+y=-x^2\cos(x)$, $y(0)=0, y(\pi)=0$	
8	$y''+x^2y'-y=x^2\exp(x)$, $y(0)=1, y(1)=e$	
9	$y''-x^2y'+y=x^2\sin x$, $y(0)=1, y(\pi)=-1$	
10	$y''-x^2y'+y=-x^2\cos(x)$, $y(0)=0, y(\pi)=0$	
11	$y''+2xy'+y=-2x\sin(x)$, $y(0)=1, y(\pi)=-1$	
12	$y''+2xy'+y=2x\cos(x)$, $y(0)=0, y(\pi)=0$	
13	$y''+2xy'-y=2x\exp(x)$, $y(0)=1, y(1)=e$	
14	$y''-2xy'+y=2x\sin(x)$, $y(0)=1, y(\pi)=-1$	
15	$y''-2xy'+y=2x\cos(x)$, $y(0)=0, y(\pi)=0$	
16	$y''+3xy'+y=-3x\sin x$, $y(0)=1, y(\pi)=-1$	
17	$y''+3xy'+y=3x\cos(x)$, $y(0)=0, y(\pi)=0$	
18	$y''+3xy'-y=3x\exp(x)$, $y(0)=1, y(1)=e$	
19	$y''-3xy'+y=-3x\sin(x)$, $y(0)=1, y(\pi)=-1$	
20	$y''-3xy'+y=3x\cos(x)$, $y(0)=0, y(\pi)=0$	
21	$y''+x^3y'+y=-x^3\sin x$, $y(0)=1, y(1)=e$	
22	$y''+x^3y'+y=x^3\cos(x)$, $y(0)=0, y(\pi)=0$	
23	$y''+x^3y'-y=x^3\exp(x)$, $y(0)=1, y(1)=e$	
24	$y''-x^3y'+y=-x^3\cos(x)$, $y(0)=1, y(\pi)=-1$	
25	$y''-x^3y'+y=x^3\sin(x)$, $y(0)=0, y(\pi)=0$	
26	$y''+4xy'+y=-4x\sin x$, $y(0)=1, y(\pi)=-1$	
27	$y''+4xy'+y=4x\cos(x)$, $y(0)=0, y(\pi)=0$	
28	$y''+4xy'-y=4x\exp(x)$, $y(0)=1, y(1)=e$	
29	$y''-4xy'+y=-4x\cos(x), y(0)=1, y(\pi)=-1$	
30	$y''-4xy'+y=4x\sin(x)$, $y(0)=0, y(\pi)=0$	

Nazariy savollar va topshiriqlar.

1.ODT uchun chegara masala nima. Chegara shartlar necha turda bo'ladi.

2.Koolokasiya, Galyorkin, Rits, EKK usullarining g'oyasi nima.

3. Kolokasiya, Galyorkin, Rits, EKK usullarida bazis funksiyalar.

4.ODT uchun ayirmali sxema nima?

Amaliy mashg'ulot №13

Mavzu: Matematik fizika tenglamalarini taqribiy yechish usullari

Mashg'ulotning maqsadi: Talabalarda matematik fizika tenglamalarini taqribiy yechish usullari ko'nikmasini shakllantirish

Mashg'ulotning jihozi:

a) Ko'rgazmali material

b) Kompyuterning dasturiy ta'minoti

c) Matematik tizimlar

Mashg'ulot texnik vositalari: Pentium IV kompyuterlari (qo'shimcha qurilmalari) bilan jihozlangan kompyuter sinfi

Mashg'ulotning borishi:

a) Auditoriya va talabalarni darsga tayyorligini aniqlash

b) Yo'qlama qilish.

c) Talabalardan o'tilgan mavzularni takrorlash

d) Yangi mavzuning bayoni: kompyuter va ko'rgazmali qurollar yordamida bayon etiladi (Aqliy hujum metodi)

e) Dars yakuni: Amaliy mashg'ulot bo'yicha har bir talabaga individual topshiriqlar beriladi.

Mavzuni qisqacha bayoni

1. Parabolik differensial tenglama uchun ayirmali sxemalar

1. PDT uchun boshlang'ich-chegara masala: $L\bar{u}=\bar{u}_t-\bar{u}_{xx}=f(x,t)$, $\bar{u}=\bar{u}(x,t)$ - aniq yechim, $0\le x\le 1,\ ,0\le t\le T$,

$$l\bar{u}=\bar{u}(x,0)=g_0(x), \quad // \quad \text{boshlang'ich shart}$$

$$l_1\bar{u}=\alpha_0\bar{u}(0,t)+\alpha_1 u_x^1(0,t)=g_1(t), \quad l_2\bar{u}=\beta_0\bar{u}(1,t)+\beta_1 u_x^1(1,t)=g_2(t) \quad // \text{ chegara shartlar.}$$

2.PDT uchun $\omega_{hr}=\{(x_k,t_j),k=0..m,j=0..n\}$ to'rda chekli ayirmali sxemalar:

1)Oshkor ayirmali sxema ($\Lambda u_h = \Lambda_{xx} u_k^j = (u_{k-1}^j - 2u_k^j + u_{k+1}^j)/h^2$.)

$$L_h^{(1)} u_h = (u_k^{j+1} - u_k^j)/\tau - \Lambda_{xx} u_k^j = f_k^j;\quad u_k^j = u(x_k, t_j) \approx \bar{u}(x_k, t_j) = \bar{u}_k^j$$

$$\alpha_0 u_0^{j+1} + \alpha_1 (u_1^{j+1} - u_0^{j+1})/h = g_1^{j+1},\ \beta_0 u_n^{j+1} + \beta_1 (u_n^{j+1} - u_{n-1}^{j+1})/h = g_2^{j+1},\quad j = 0,\dots,n.$$

2)Sof oshkormas ayirmali sxema

$$L_h^{(2)} u_h = (u_k^{j+1} - u_k^j)/\tau - \Lambda_{xx} u_k^{j+1} = f_k^{j+1};$$

$$u_k^0 = g_0(x_k),\ k = 0,\dots,m;$$

$$\alpha_0 u_0^{j+1} + \alpha_1 (u_1^{j+1} - u_0^{j+1})/h = g_1^{j+1},\ \beta_0 u_n^{j+1} + \beta_1 (u_n^{j+1} - u_{n-1}^{j+1})/h = g_2^{j+1},\quad j = 0,\dots,n$$

3) Krank-Nikolsonning uqori tartibli approksimasiyali sxemasi

$$L_h^{(3)} u_h = (u_k^{j+1} - u_k^j)/\tau - 0.5\Lambda(u_k^j + u_k^{j+1}) = 0.5(f_k^j + f_k^{j+1}) = f_k^{j+0.5}$$

Algoritmlar: $u_k^{j+1} = u_k^j + \tau\Lambda_{xx} u_k^j + \tau f_k^j;\quad (E - \tau\Lambda_{xx}) u_k^{j+1} = u_k^j + \tau f_k^{j+1}$,

$$(E - 0.5\tau\Lambda_{xx}) u_k^{j+1} = (E + 0.5\tau\Lambda_{xx}) u_k^j + 0.5\tau f_k^{j+1/2}.$$

$$u_k^0 = g_0(x_k),\ k = 0,\dots,m;$$

$$\alpha_0 u_0^{j+1} + \alpha_1 (u_1^{j+1} - u_0^{j+1})/h = g_1^{j+1},\ \beta_0 u_n^{j+1} + \beta_1 (u_n^{j+1} - u_{n-1}^{j+1})/h = g_2^{j+1},\quad j = 0,\dots,n$$

3. Approksimasiyani tekshirish:

$$R_h^{(1)} = L_h^{(1)} u_h - f_h = \{Lu(x_k, t_j) - f(x_k, t_j)\} = 0(\tau + h^2),$$

$$R_h^{(2)} = L_h^{(2)} u_h - f_h = \{Lu(x_k, t_j) - f(x_k, t_j)\} = 0(\tau + h^2),$$

$$R_h^{(3)} = L_h^{(3)} u_h - f_h = \{Lu(x_k, t_j) - f(x_k, t_j)\} = 0(\tau^2 + h^4).$$

Chegara shartlar $0(\tau + h)$ xatolik bilan approksimasiya qilinyapti. Ularni $0(\tau + h^2)$xatolik bilan ham approksimasiya qilinishi mumkin [23-25].

4.Turg'unlikni tekshirish:a)oshkor sxemaning turg'unlik sharti $\tau / h^2 \le 1/2$;

b) oshkormas sxemalar shartsiz (absolyut) turg'un .

5. Fillipov teoremasi. Ayirmali sxemaning approksimasiyasi va turg'unligidan uning yaqinlashishi kelib chiqadi.

PDT uchun ayirmali sxemani Mathcadda yechishni tashkil etish.

PDT uchun ushbu chegara masalani qaraymiz:

$ua(x,t) := (x - x^2)e^t$, $f(x,t) = (x - x^2 + 2)e^t$, $g0(x) = (x - x^2)$, $g_1(t) = 0, g2(t) = 0$

// belgidan so'ng izohlarni keltiramiz. Izoh dasturning bajarilishiga ta'sir etmaydi.

A) Pdesolve ichki funksiya yordamida yechish.

Mathcad dasturida quyidagi komandalarni kiritamiz:

$a := 0$ $b := 1$ $L := b - a$ $T := 0.05$ //Sohani berish

$m := 10$ $n := 5$ $i := 0..m$ $j := 0..n$, $h := L/m$ $\tau := T/n$ $x_i := a + ih$ $t_j := j\tau$ // *to'r*

$g0(x) := x - x^2$ $f(x,t) := (x - x^2 + 2)e^t$ $g1(t) := 0$ $g2(t) := 0$ // *Berilganlar*

Given $u_t(x,t) = u_{xx}(x,t) + f(x,t)$ (barobar yo'g'on) //*Tenglama*

$u(x,0) = g0(x)$ $u(0,t) = g1(t)$ $u(1,t) = g2(t)$ // *Chegara shartlar*

$u := Pdesolve\left[u, x, \begin{pmatrix} a \\ b \end{pmatrix}, t, \begin{pmatrix} 0 \\ T \end{pmatrix}, m, n\right]$ $u_{i,j} := u(x_i, t_j)$ // *Ichki funksiyaga mur.-t*

$ua(x,t) := (x - x^2)e^t$ $ua_{i,j} := ua(x_i, t_j)$ //*Aniq yechim va qiymatlar*

// *Taqribiy yechim jadvalini chiqaramiz:*

13.1.1-jadval

$u^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0.089	0.158	0.207	0.237	0.247	0.237	0.207	0.158	0.089
1	0	0.09	0.16	0.209	0.239	0.249	0.239	0.209	0.16	0.09
2	0	0.091	0.161	0.212	0.242	0.252	0.242	0.212	0.161	0.091
3	0	0.092	0.163	0.214	0.244	0.254	0.244	0.214	0.163	0.092
4	0	0.093	0.164	0.216	0.247	0.257	0.247	0.216	0.164	0.093
5	0	0.093	0.166	0.218	0.249	0.26	0.249	0.218	0.166	0.093

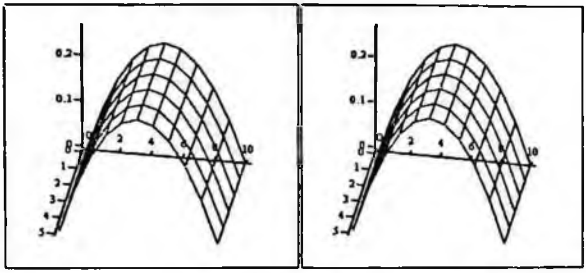
//*Aniq yechim jadvalini chiqaramiz:*

13.1.2-jadval

$ua^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09
1	0	0.091	0.162	0.212	0.242	0.253	0.242	0.212	0.162	0.091
2	0	0.092	0.163	0.214	0.245	0.255	0.245	0.214	0.163	0.092
3	0	0.093	0.165	0.216	0.247	0.258	0.247	0.216	0.165	0.093
4	0	0.094	0.167	0.219	0.25	0.26	0.25	0.219	0.167	0.094
5	0	0.095	0.168	0.221	0.252	0.263	0.252	0.221	0.168	0.095

//*Aniq va taqribiy yechim grafikini chiqaramiz:*



u^T $\qquad\qquad$ ua^T

13.1-rasm

B) PDT uchun oshkor ayirmali sxemani Mathcadda yechish.

Mathcad oynasida quyidagi komandalarni yozib chiqamiz:

$a:=0\ b:=1\ L:=b-a\ T:=0.05$ // Soha

$m:=10\ n:=5\ i:=0..m\ j:=0..n, h:=L/m\ \tau:=T/n\ x_i:=a+ih\ t_j:=j\tau$ // *to'r*

$g0(x):=x-x^2\ \ f(x,t):=(x-x^2+2)e^t\ \ g1(t):=0\ \ g2(t):=0$ // *Berilganlar*

$u_{i,0}:=g0(x_i)\ u_{0,j}:=g1(t_j)\ u_{m,j}:=g2(t_j)$ // *Qo'shimcha shartlar*

// u_k *qiymatlarni qatlam bo'yicha hisoblash va chiqarish*

$i:=1..m-1\ \ j:=0..n-1\ \ u_{i,j+1}:=r(u_{i-1,j}+u_{i+1,j})+(1-2r)u_{i,j}+\tau f(x_i,t_j)\quad u^T=$

$ua(x,t):=(x-x^2)e^t\quad ua_{i,j}:=ua(x_i,t_j)\ \ ua^T=$ // *Aniq yechim, qiymatlari*

C) Sof oshkormas ayirmali sxemani Mathcadda yechish.

Mathcad oynasida quyidagi komandalarni yozib chiqamiz:

$a:=0\ b:=1\ L:=b-a\ T:=0.05$ // soha

$m:=10\ n:=5\ i:=0..m\ j:=0..n, h:=L/m\ \tau:=T/n\ x_i:=a+ih\ t_j:=j\tau$ // *to'r*

$ua(x,t):=(x-x^2)e^t\quad ua_{i,j}:=ua(x_i,t_j)$ // *Aniq yechim va uning jadvali*

$g0(x):=x-x^2\ \ f(x,t):=(x-x^2+2)e^t\ \ g1(t):=0\ \ g2(t):=0$ // *Berilganlar*

$u_{i,0}:=u0(x_i)\ u_{0,j}:=g1(t_j)\ u_{m,j}:=g2(t_j)$ // *Qo'shimcha shartlar*

$A_{0,0}:=1\ i:=1..m\ A_{0,i}:=0\quad A_{m,m}:=1\ i=0..m-1\ A_{m,i}:=0$

$i:=1..m-1\ A_{i,i-1}:=-r\ \ A_{i,i+1}:=-r\ \ A_{i,i}:=1+2r$ // *CHAS matrisasi*

$i:=1..m-1\ j:=0..n-1\ \ d_{0,j}:=g1(t_{j+1})\ \ d_{m,j}:=g2(t_{j+1})\ \ d_{i,j}:=\tau f_{i,j+1}$ // *Berilganlar*

// u_k qiymatlarni qatlam bo'yicha hisoblash va chiqarish

$j := 0..n-1 \quad u^{\langle j+1\rangle} := A^{-1}(d^{\langle j\rangle} + u^{\langle j\rangle})$ *// CHAS matrisasini nazorat uchun chiqarish*

13.1.3-jadval

A=

A =

	0	1	2	3	4	5	6	7	8	9	10
0	1	0	0	0	0	0	0	0	0	0	0
1	-0.5	2	-0.5	0	0	0	0	0	0	0	0
2	0	-0.5	2	-0.5	0	0	0	0	0	0	0
3	0	0	-0.5	2	-0.5	0	0	0	0	0	0
4	0	0	0	-0.5	2	-0.5	0	0	0	0	0
5	0	0	0	0	-0.5	2	-0.5	0	0	0	0
6	0	0	0	0	0	-0.5	2	-0.5	0	0	0
7	0	0	0	0	0	0	-0.5	2	-0.5	0	0
8	0	0	0	0	0	0	0	-0.5	2	-0.5	0
9	0	0	0	0	0	0	0	0	-0.5	2	-0.5
10	0	0	0	0	0	0	0	0	0	0	1

u^T = //Taqribiy yechim u_k jadvali

13.1.4-jadval

u^T =

	0	1	2	3	4	5	6	7	8	9
0	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09
1	0	0.09	0.161	0.211	0.241	0.251	0.241	0.211	0.161	0.09
2	0	0.091	0.162	0.212	0.242	0.253	0.242	0.212	0.162	0.091
3	0	0.091	0.162	0.213	0.244	0.254	0.244	0.213	0.162	0.091
4	0	0.092	0.163	0.214	0.245	0.255	0.245	0.214	0.163	0.092
5	0	0.092	0.164	0.215	0.246	0.256	0.246	0.215	0.164	0.092
6	0	0.093	0.165	0.216	0.247	0.258	0.247	0.216	0.165	0.093
7	0	0.093	0.166	0.217	0.249	0.259	0.249	0.217	0.166	0.093
8	0	0.094	0.167	0.219	0.25	0.26	0.25	0.219	0.167	0.094
9	0	0.094	0.167	0.22	0.251	0.262	0.251	0.22	0.167	0.094
10	0	0.095	0.168	0.221	0.252	0.263	0.252	0.221	0.168	0.095

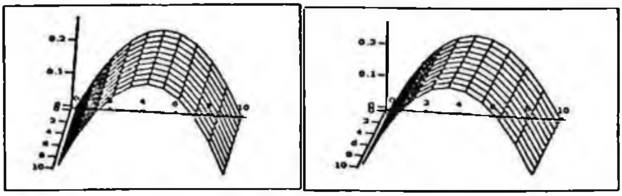
$\bar{u}_k$ *// Aniq yechim jadvali*

13.1.4-jadval

ua^T =

	0	1	2	3	4	5	6	7	8	9
0	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09
1	0	0.09	0.161	0.211	0.241	0.251	0.241	0.211	0.161	0.09
2	0	0.091	0.162	0.212	0.242	0.253	0.242	0.212	0.162	0.091
3	0	0.091	0.162	0.213	0.244	0.254	0.244	0.213	0.162	0.091
4	0	0.092	0.163	0.214	0.245	0.255	0.245	0.214	0.163	0.092
5	0	0.092	0.164	0.215	0.246	0.256	0.246	0.215	0.164	0.092
6	0	0.093	0.165	0.216	0.247	0.258	0.247	0.216	0.165	0.093
7	0	0.093	0.166	0.217	0.249	0.259	0.249	0.217	0.166	0.093
8	0	0.094	0.167	0.219	0.25	0.26	0.25	0.219	0.167	0.094
9	0	0.094	0.167	0.22	0.251	0.262	0.251	0.22	0.167	0.094
10	0	0.095	0.168	0.221	0.252	0.263	0.252	0.221	0.168	0.095

//Yechimlarning grafiklarini chiqarish



u^T ua^T

13.1.2-rasm

Izoh. 0- va m-tenglamalardagi nomalumlar 1-tur chegara shartlarda berilgan. Biz ulardan foydalanib CHATS tenglamalar sonini 2 taga kamaytirmadik, ularni tenglama sifatida o'z o'rnida qoldirdik. CHunki, 2- va 3- tur chegara shartlarning approksimasiyalarini albatta, 0- va m-tenglamalar qilib yozib olinadi.

D) PDT uchun Krank- Nikolson ayirmali sxemasini Mathcadda yechish

//PDT uchun Kranka- Nikolsona ayirmali sxemasi

$u_t = u_{xx} + f(x,t)$ $ut(x,t) := (x - x^2)e^t$ $f(x,t) := (x - x^2 + 2)e^t$ *// PDT*

$g1(t) := ut(0,t)$ $g2(t) := ut(1,t)$ $g0(x) := ut(x,0)$ $ff(x,t) := (x - x^2 + 2)e^t$ *// Berilganlar*

$x0 := 0$ $xm := 1$ $m := 10$ $h := (xm - x0)/m$ $i := 0..m$ $x_i := x0 + ih$ *// To'r*

$t0 := 0$ $tn := 0.1$ $n := 10$ $\tau := (tn - t0)/n$ $j := 0..n$ $t_j := t0 + j\tau$ $\mu := \tau/h^2$ $\tau = 0.01$ *//To'r*

$i := 0..m$ $j := 0..n$ $f_{i,j} := f(x_i, t_j)$ $ut_{i,j} := ut(x_i, t_j)$ $u_{i,0} := u0(x_i)$ $\mu = 1$ *//Qiymatlar*

$\gamma := 2(1+\mu)$ $\gamma = 4$ $\gamma 1 := 2(1-\mu)$ *// Parametrlar*

// ayirmali sxema matrisasini yaratish

$A_{0,0} := 1$ $k := 1..m$ $A_{0,k} := 0$ $B_{0,0} := 0$ $k := 1..m$ $B_{0,k} := 0$

$A_{m,m} := 1$ $k := 1..m-1$ $A_{m,k} := 0$ $B_{m,m} := 0$ $k := 1..m-1$ $B_{m,k} := 0$

$k := 1$ $A_{k,k-1} := -1$ $A_{k,k} := \gamma$ $A_{k,k+1} := -1$ $B_{k,k-1} := 1$ $B_{k,k} := \gamma 1$ $B_{k,k-1} := 1$

//ayirmali sxemaning o'ng tomonini yaratish

$j := 0..n-1$ $d_{0,j+1} := 0$ $d_{m,j+1} := 0$ $i := 1..m-1$ $d_{i,j} := 2h^2 ff(x_i, t_j + \tau/2)$

//ayirmali sxemani qatlamlar bo'yicha chiqarish

$j := 0..n-1$ $u^{\langle j+1 \rangle} := lsolve(A, d^{\langle j \rangle} + Bu^{\langle j \rangle})$

//Aniq va taqribiy yechim jadvallarini , grafiklarni aniqroq chiqarish

13.1.5-jadval

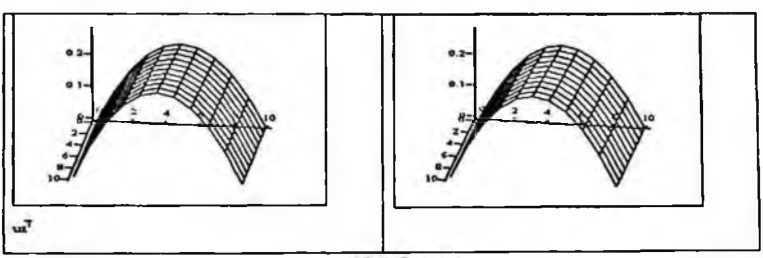
$ut^T =$

	0	1	2	3	4	5	6
0	0	0.09	0.16	0.21	0.24	0.25	0.24
1	0	0.0909	0.16161	0.21211	0.24241	0.25251	0.24241
2	0	0.09182	0.16323	0.21424	0.24485	0.25505	0.24485
3	0	0.09274	0.16487	0.2164	0.24731	0.25761	0.24731
4	0	0.09367	0.16653	0.21857	0.24979	0.2602	0.24979
5	0	0.09461	0.1682	0.22077	0.25231	0.26282	0.25231
6	0	0.09557	0.16989	0.22299	0.25484	0.26546	0.25484
7	0	0.09653	0.1716	0.22523	0.2574	0.26813	0.2574
8	0	0.0975	0.17333	0.22749	0.25999	0.27082	0.25999
9	0	0.09848	0.17507	0.22978	0.2626	0.27354	0.2626
10	0	0.09947	0.17683	0.23209	0.26524	0.27629	...

13.1.6-jadval

$u^T =$

	0	1	2	3	4	5	6
0	0	0.09	0.16	0.21	0.24	0.25	0.24
1	0	0.0909	0.16161	0.21211	0.24241	0.25251	0.24241
2	0	0.09182	0.16323	0.21424	0.24485	0.25505	0.24485
3	0	0.09274	0.16487	0.21639	0.24731	0.25761	0.24731
4	0	0.09367	0.16653	0.21857	0.24979	0.2602	0.24979
5	0	0.09461	0.1682	0.22077	0.2523	0.26282	0.2523
6	0	0.09556	0.16989	0.22298	0.25484	0.26546	0.25484
7	0	0.09653	0.1716	0.22523	0.2574	0.26813	0.2574
8	0	0.0975	0.17332	0.22749	0.25999	0.27082	0.25999
9	0	0.09847	0.17507	0.22977	0.2626	0.27354	0.2626
10	0	0.09946	0.17683	0.23208	0.26524	0.27629	...



13.1.3-rasm

Uqori tartibli aniqlikdagi ayirmali sxemaning afzalligi ko'rinib turibdi: natijalarda farq faqat verguldan so'ng 5 –raqamda sezilmoqda. Natijalar Pdesolve ichki funksiyasini bilan mos.

Individual topshiriqlar.

Differensial tenglamalar uchun oshkor va oshkormas sxemalar tuzilsin. Ular Mathcad dasturida ichki funksiyalar va algoritm tuzib echilsin, natijalarning yaqinligiga yerishilsin.

PDT uchun boshlang'ich-chegara masala: $u_t - u_{xx} = f(x,t)$, $u(x,0) = g(x)$, $u(0,t) = \mu_1(t), u(1,t) = \mu_2(t)$.

13.1.7-jadval

№	$u(x,t)$	$u(x,0)$	$u(0,t)$	$u(1,t)$	$f(x,t)$
1-5	$x(1-x)\sin nt$	0	0	0	$f(x,t) = u_t(.) - u_{xx}(.)$
6-10	$x(1-x)\cos nt$	$x(1-x)$	0	0	$f(x,t) = u_t(.) - u_{xx}(.)$
11-15	$x(1-x)\exp nt$	$x(1-x)$	0	0	$f(x,t) = u_t(.) - u_{xx}(.)$
16-20	$x(1-x)shnt$	0	0	0	$f(x,t) = u_t(.) - u_{xx}(.)$
21-25	$x(1-x)chnt$	$x(1-x)$	0	0	$f(x,t) = u_t(.) - u_{xx}(.)$
26-30	$x(1-x)e^{-nt}$	$x(1-x)$	0	0	$f(x,t) = u_t(.) - u_{xx}(.)$
31-35	$\alpha x^3 + \beta t^2$	αx^3	βt^2	$1+\beta t^2$	$f(x,t) = u_t(.) - u_{xx}(.)$
36-40	$\alpha x^3 + \beta t^3$	αx^3	βt^3	$1+\beta t^3$	$f(x,t) = u_t(.) - u_{xx}(.)$

Nazariy savollar va topshiriqlar.

1.PDT nima?

2.CHegara shart nechta va necha turda?

3.PDT uchun oshkor sxema nima?

4. PDT uchun Krank-Nikolson sxemasi nima va u qanday echiladi?

5. PDT uchun oshkor sxema approksimasiyasi qanday?

6. Boshlang'ich shart approksimasiyasi qanday?

2. GDT uchun chekli ayirmali sxemalar.

GDT uchun chekli ayirmali sxemalarning approksimasiyasi, turg'unligi, yaqinlashishi, algoritmi.

Asosiy formulalar:

1. GDT uchun boshlang'ich-chegara masala.

$Lu = u_{tt} - u_{xx} = f(x,t)$, $\quad 0 < x < 1,\ 0 < t < 1, u(x,0) = u_0(x) = u0(x),\ u'(x,0) = u_1(x) = u1(x)$

$$l_1\bar{u}=\alpha_0\bar{u}(0,t)+\alpha_1 u_x'(0,t)=g_1(t),\quad l_2\bar{u}=\beta_0\bar{u}(1,t)+\beta_1 u_x'(1,t)=g_2(t), 0\le t\le T$$

2. GDT uchun ayirmali sxemada boshlang'ich va chegara shartlar approksimasiyasi .

$$\{u_k^{j-1}-2u_k^j+u_k^{j+1}\}/\tau^2-\Lambda_{xx}u_k^j=f_k^j,\quad k=1,...,m-1,\quad j=1,...,n-1,$$

$$u_k^0=u0_k, u_k^1=u0_k+\tau u1_k+\frac{\tau^2}{2}\{f_k^0+\Lambda_{xx}u0_k\},\ k=1,...,m-1 \text{ ,yoki } u_k^1=u0_k+\tau u1_k\ ,$$

$$\alpha_0 u_0^{j+1}+\alpha_1(u_1^{j+1}-u_0^{j+1})/h=g_1^{j+1},\ \beta_0 u_n^{j+1}+\beta_1(u_n^{j+1}-u_{n-1}^{j+1})/h=g_2^{j+1},\quad j=0,...,n.$$

3. Approksimasiya.

$$R_h=L_h u_h-f_h=0(\tau^2+h^2), r_h=l_h u_h-g_h=0(\tau)\text{ , yoki}$$

$$R_h=L_h u_h-f_h=0(\tau^2+h^2), r_h=l_h u_h-g_h=0(\tau^2).$$

CHegara shartlar $0(h)$ xatolik bilan approksimasiya qilinyapti. Ularni $0(h^2)$ xatolik bilan ham approksimasiya qilinishi mumkin [23-25].

4. Turg'unlik : oshkor sxema shartli turg'un: $\tau\le h$, oshkormas ayirmali sxemada absolyut turg'un.

5.Mathcadda algoritm

1)Oshkor ayirmali sxema: $u^{<j+1>}=Bu^{<j>}+\tau^2 f^{<j>}, j=1..n-1,$

$$u^{<0>}=\{u0_i\}, u^{<1>}=\{u0_i+\tau u1_i\}$$

2) Sof oshkormas ayirmali sxema:

$$Au^{<j+1>}=-u^{<j-1>}+2u^{<j>}+\tau^2 f^{<j+1>}, u^{<0>}=\{u0_i\}, u^{<1>}=\{u0_i+\tau u1_i\}$$

3) Krank-Nikolson oshkormas ayirmali sxemasi:

$$Au^{<j+1>}=-Bu^{<j-1>}+2u^{<j>}+\tau^2 f^{<j+1>}, u^{<0>}=\{u0_i\}, u^{<1>}=\{u0_i+\tau u1_i\}$$

6. Fillipov teoremasi. Ayirmali sxema approksimasiyasi va turg'unligidan uning yaqinlashishi kelib chiqadi.

GDT uchun CHASni Mathcadda yechishni tashkil etish.

GDT uchun ushbu boshlang'ich chegara masalani qaraymiz:

$$Lu=u_{tt}-u_{xx}=f(x,t)=(x-x^2+2)e^t,\ u=u(x,t), a\le x\le b, 0\le t\le T$$

$$u(x,0)=u_0(x)=(x-x^2), u_t'(x,0)=u_1(t)=(x-x^2)\quad u(a,t)=g_1(t)=0, u(b,t)=g_2(t)=0\ .$$

A) ayirmali sxemani ichki funksiya Pdesolve bilan yechish.

Ushbu komandalarni Matcadda yozib chiqamiz:

$a:=0\ b:=1\ \ L:=b-a\ T:=0.0005$ // *soha*

$m:=10\ n:=10\ i:=0..m\ j:=0..n, h:=L/m\quad \tau:=T/n\quad x_i:=a+ih\ t_j:=j\tau$ // *to'r*

$ua(x,t):=ua(x,t)\ \ ua_{i,j}:=ua(x_i,t_j)$ // *aniq yechim* $ua(x,t)$ *va karkas* $\bar{u}_h$

$u0(x):=x-x^2\ \ u1(x):=x-x^2\ \ f(x,t):=(x-x^2+2)e^t\ \ g1(t):=0\ \ g2(t):=0$ // *BCHSH*

Given $w_t(x,t):=v(x,t)\ \ v_t(x,t):=w_{xx}(x,t)+f(x,t)$ // *GDT*

$w(x,0):=u0(x)\ \ v(x,0):=u1(x)\ \ w(0,t):=g1(t)\ \ w(1,t):=g2(t)$ // *berilgan*

$\begin{pmatrix} w \\ v \end{pmatrix}:=Pdesolve\left[\begin{pmatrix} w \\ v \end{pmatrix}, x, \begin{pmatrix} a \\ b \end{pmatrix}, t, \begin{pmatrix} 0 \\ T \end{pmatrix}, n, n\right]\quad u_{i,j}:=w(x_i,t_j)$ // *ichki funksiya*

// *Aniq yechim va taqribiy yechim jadvalini chiqaramiz:*

13.2.1-jadval

$u^T =$

	0	1	2	3	4	5	6	7	8	9	10
0	0	0.0889	0.158	0.2074	0.237	0.2469	0.237	0.2074	0.158	0.0889	0
1	0	0.0898	0.1596	0.2095	0.2394	0.2494	0.2394	0.2095	0.1596	0.0898	0
2	0	0.0907	0.1612	0.2116	0.2418	0.2519	0.2418	0.2116	0.1612	0.0907	0
3	0	0.0916	0.1628	0.2137	0.2443	0.2544	0.2443	0.2137	0.1628	0.0916	0
4	0	0.0925	0.1645	0.2159	0.2467	0.257	0.2467	0.2159	0.1645	0.0925	0
5	0	0.0934	0.1661	0.218	0.2492	0.2596	0.2492	0.218	0.1661	0.0934	0

13.2.2-jadval

$ua^T =$

	0	1	2	3	4	5	6	7	8	9	10
0	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
1	0	0.0909	0.1616	0.2121	0.2424	0.2525	0.2424	0.2121	0.1616	0.0909	0
2	0	0.0918	0.1632	0.2142	0.2448	0.2551	0.2448	0.2142	0.1632	0.0918	0
3	0	0.0927	0.1649	0.2164	0.2473	0.2576	0.2473	0.2164	0.1649	0.0927	0
4	0	0.0937	0.1665	0.2186	0.2498	0.2602	0.2498	0.2186	0.1665	0.0937	0
5	0	0.0946	0.1682	0.2208	0.2523	0.2628	0.2523	0.2208	0.1682	0.0946	0

V) Oshkor shartli turg'un ayirmali sxema .

Ushbu komandalarni Matcadda yozib chiqamiz:

$a:=0\ b:=1\ \ L:=b-a\ T:=0.05\ m:=10\ n:=5\ i:=0..m\ j:=0..n$ // *soha*

$h:=L/m\quad \tau:=T/n\quad x_i:=a+ih\ t_j:=j\tau\quad r:=\tau^2/h^2$ // *to'r*

$ua(x,t):=(x-x^2)e^t, ua_{i,j}:=ua(x_i,t_j)$ // *aniq yechim* $ua(x,t)$ *va karkas*

$u0(x):=x-x^2\ \ u1(x):=x-x^2\ \ f(x,t):=(x-x^2+2)e^t\ \ g1(t):=0\ \ g2(t):=0$ // *BvaCHSH*

$u_{i,0} := u0(x_i)$ $u_{i,1} := u_{i,0} + \tau u1(x_i)$ $u_{0,j} := g1(t_j)$ $u_{m,j} := g2(t_j)$ *// oshkor AS*

$i := 1..m-1$ $j := 1..n-1$ $u_{i,j+1} := r(u_{i-1,j} + u_{i+1,j}) + 2(1-r)u_{i,j} + \tau^2 f(x_i, t_j) - u_{i,j-1}$

//Aniq va taqribiy yechim jadvalini chiqaramiz:

//Farqlar o'nli razıyadlar oshirilsagina seziladi.

13.2.3-jadval

$u^T =$

	0	1	2	3	4	5	6	7	8	9	10
0	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
1	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
2	0	0.09	0.16	0.21	0.24	0.2501	0.24	0.21	0.16	0 09	0
3	0	0.09	0.16	0.2101	0.2401	0.2501	0.2401	0 2101	0.16	0.09	0
4	0	0.09	0.1601	0.2101	0.2401	0.2501	0.2401	0 2101	0.1601	0.09	0
5	0	0.09	0.1601	0.2101	0.2401	0.2501	0.2401	0.2101	0.1601	0.09	0
6	0	0.0901	0.1601	0.2101	0.2401	0.2501	0.2401	0.2101	0.1601	0.0901	0
7	0	0.0901	0.1601	0.2101	0.2402	0.2502	0.2402	0.2101	0.1601	0.0901	0
8	0	0 0901	0.1601	0.2102	0.2402	0.2502	0.2402	0.2102	0.1601	0.0901	0
9	0	0 0901	0 1601	0.2102	0.2402	0.2502	0.2402	0 2102	0 1601	0.0901	0
10	0	0.0901	0.1602	0.2102	0.2402	0.2502	0.2402	0.2102	0.1602	0.0901	0

13.2.4-jadval

$ua^T =$

	0	1	2	3	4	5	6	7	8	9	10
0	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
1	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
2	0	0.09	0.16	0.21	0.24	0.2501	0.24	0.21	0.16	0.09	0
3	0	0.09	0.16	0.2101	0.2401	0.2501	0.2401	0.2101	0.16	0.09	0
4	0	0.09	0.1601	0.2101	0.2401	0.2501	0.2401	0.2101	0.1601	0.09	0
5	0	0.09	0.1601	0.2101	0.2401	0.2501	0.2401	0.2101	0.1601	0.09	0
6	0	0.0901	0.1601	0.2101	0.2401	0.2502	0.2401	0.2101	0.1601	0.0901	0
7	0	0.0901	0.1601	0.2101	0.2402	0.2502	0.2402	0.2101	0.1601	0.0901	0
8	0	0.0901	0.1601	0.2102	0.2402	0.2502	0.2402	0.2102	0.1601	0.0901	0
9	0	0.0901	0.1601	0 2102	0.2402	0.2502	0.2402	0.2102	0.1601	0.0901	0
10	0	0.0901	0.1602	0.2102	0.2402	0.2503	0.2402	0.2102	0.1602	0.0901	0

C) Sof oshkormas absolyut turg'un AS.

Ushbu komandalarni Matcadda yozib chiqamiz:

$a := 0$ $b := 1$ $L := b - a$ $T := 0.0005$ $m := 10$ $n := 10$ $i := 0..m$ $j := 0..n$ *// soha*

$h := L/m$ $\tau := T/n$ $x_i := a + ih$ $t_j := j\tau$ $ut_{i,j} := ut(x_i, t_j)$ $r := \tau^2/h^2$ *// to'r*

$u0(x) := x - x^2$ $u1(x) := x - x^2$ $f(x,t) := (x - x^2 + 2)e^t$ $g1(t) := 0$ $g2(t) := 0$ *// BvaCHSH*

$u_{i,0} := u0(x_i)$ $u_{i,1} := u_{i,0} + \tau u1(x_i)$ $u_{0,j} := g1(t_j)$ $u_{m,j} := g2(t_j)$ *// qo'shimcha shartlar*

// AS matrisasi va algoritm qadamini berish

$A_{0,0} := 1$ $i := 1..m$ $A_{0,i} := 0$

$i := 1..m-1$ $A_{i,i-1} := -r$ $A_{i,i+1} := -r$ $A_{i,i} := 1 + 2r$

$i := 0..m-1$ $A_{m,i} := 0$ $A_{m,m} := 1$

$j := 1..n-1 \quad u^{<j+1>} := A^{-1}(-u^{<j-1>} + \tau^2 f^{<j+1>} + 2u^{<j>})$

Nazorat uchun matrisa, boshlang'ich shartlarni chiqarish:

13.2.5-jadval

A =

	0	1	2	3	4	5	6	7	8	9	10
0	1	0	0	0	0	0	0	0	0	0	0
1	-0.005	1.01	-0.005	0	0	0	0	0	0	0	0
2	0	-0.005	1.01	-0.005	0	0	0	0	0	0	0
3	0	0	-0.005	1.01	-0.005	0	0	0	0	0	0
4	0	0	0	-0.005	1.01	-0.005	0	0	0	0	0
5	0	0	0	0	-0.005	1.01	-0.005	0	0	0	0
6	0	0	0	0	0	-0.005	1.01	-0.005	0	0	0
7	0	0	0	0	0	0	-0.005	1.01	-0.005	0	0
8	0	0	0	0	0	0	0	-0.005	1.01	-0.005	0
9	0	0	0	0	0	0	0	0	-0.005	1.01	-0.005
10	0	0	0	0	0	0	0	0	0	0	1

// 0- va 1-qatlam qiymatlari

13.2.6-jadval

$u^{\langle 0 \rangle T} =$

	0	1	2	3	4	5	6	7	8	9	10
0	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0

$u^{\langle 1 \rangle T} =$

	0	1	2	3	4	5	6	7	8	9	10
0	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0

//Aniq va taqribiy yechim jadvalini chiqaramiz:

13.2.7-jadval

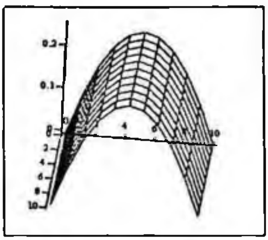
$u^T =$

	0	1	2	3	4	5	6	7	8	9	10
0	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
1	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
2	0	0.0899	0.1599	0.2099	0.2399	0.2499	0.2399	0.2099	0.1599	0.0899	0
3	0	0.0897	0.1597	0.2097	0.2397	0.2497	0.2397	0.2097	0.1597	0.0897	0
4	0	0.0894	0.1594	0.2094	0.2394	0.2494	0.2394	0.2094	0.1594	0.0894	0
5	0	0.089	0.159	0.2091	0.2391	0.2491	0.2391	0.2091	0.159	0.089	0
6	0	0.0886	0.1585	0.2086	0.2386	0.2486	0.2386	0.2086	0.1585	0.0886	0
7	0	0.088	0.158	0.208	0.238	0.248	0.238	0.208	0.158	0.088	0
8	0	0.0873	0.1573	0.2073	0.2373	0.2473	0.2373	0.2073	0.1573	0.0873	0
9	0	0.0866	0.1565	0.2065	0.2365	0.2465	0.2365	0.2065	0.1565	0.0866	0
10	0	0.0858	0.1556	0.2056	0.2356	0.2456	0.2356	0.2056	0.1556	0.0858	0

13.2.8-jadval

	0	1	2	3	4	5	6	7	8	9	10
0	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
1	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
2	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
3	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
4	0	0.09	0.16	0.21	0.24	0.2501	0.24	0.21	0.16	0.09	0
5	0	0.09	0.16	0.2101	0.2401	0.2501	0.2401	0.2101	0.16	0.09	0
6	0	0.09	0.16	0.2101	0.2401	0.2501	0.2401	0.2101	0.16	0.09	0
7	0	0.09	0.1601	0.2101	0.2401	0.2501	0.2401	0.2101	0.1601	0.09	0
8	0	0.09	0.1601	0.2101	0.2401	0.2501	0.2401	0.2101	0.1601	0.09	0
9	0	0.09	0.1601	0.2101	0.2401	0.2501	0.2401	0.2101	0.1601	0.09	0
10	0	0.09	0.1601	0.2101	0.2401	0.2501	0.2401	0.2101	0.1601	0.09	0

// Aniq va taqribiy yechim grafiklarini chiqaramiz:



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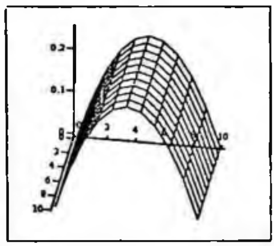


uu^T

13.2.1-rasm

D) Krank-Nikolsonning absolyut turg'un ayirmali sxema bilan eksperiment

Ushbu komandalarni Matcadda yozib chiqamiz:

GDT $u_{tt}=u_{xx}+f(x,t)$ **Krank-Nikolson ayirmali sxemasi**

// $Au^{<j+1>}=Bu^{<j-1>}+4u^{<j>}+2\tau^2 f^{<j+1/2>}$

$ut(x,t):=(x-x^2)e^t \quad f(x,t):=(x-x^2+2)e^t \quad ff(x,t):=(x-x^2+2)e^t \quad g1(t):=ut(0,t) \quad g2(t):=ut(1,t)$

$u0(x):=ut(x,0) \quad dut(x,t):=dut(x,t)/dt \quad u1(x):=dut(x,0)\rightarrow x-x^2$

$x0:=0 \quad xm:=1 \quad m:=10 \quad h:=(xm-x0)/m \quad i:=0..m \quad x_i:=x0+ih$

$t0:=0 \quad tn:=1 \quad n:=10 \quad \tau:=(tn-t0)/n \quad j:=0..n \quad t_j:=t0+j\tau \quad r:=(\tau/h)^2$

$i:=0..m \quad j:=0..n \quad f_{i,j}:=f(x_i,t_j) \quad ut_{i,j}:=ut(x_i,t_j) \quad ff_{i,j}:=ff(x_i,t_j+\tau/2) \quad u_{i,0}:=u0(x_i)$

$i:=1..m-1 \quad u_{i,1}:=u_{i,0}+(r/2)(u_{i-1,0}-2u_{i,0}+u_{i+1,0})$

// CHAS matrisasini yaratish

$\gamma:=2r \quad i:=0 \quad j:=0..m \quad A_{i,j}:=if(j=0,1,0) \quad B_{i,j}:=0$

$i:=1..m-1 \quad A_{i,j}:=if[(j\neq i-1)*(j\neq i+1),0,-r] \quad A_{i,i}:=2(1+r)$

$i:=1..m-1 \quad B_{i,j}:=if[(j\neq i-1)*(j\neq i+1),0,r] \quad B_{i,i}:=-2(1+r)$

$i := m\ \ A_{i,j} := if(j = m, 1, 0)\ \ B_{i,j} := 0$

// Chegara shartlar va o'ng tomon

$j := 0..n\ \ d_{0,j} := g1(t_j)\ \ d_{m,j} := g2(t_j)\ \ i := 1..m-1\ \ d_{i,j} := if[(i=0)*(i=m), 0, 2\tau^2 ff_{i,j}]$

13.2.9-jadval

$d^T =$

	0	1	2	3	4	5	6	7	8	9	10
0	0	0.044	0.045	0.046	0.047	0.047	0.047	0.046	0.045	0.044	0
1	0	0.049	0.05	0.051	0.052	0.052	0.052	0.051	0.05	0.049	0
2	0	0.054	0.055	0.057	0.058	0.058	0.058	0.057	0.055	0.054	0
3	0	0.059	0.061	0.063	0.064	0.064	0.064	0.063	0.061	0.059	0
4	0	0.066	0.068	0.069	0.07	0.071	0.07	0.069	0.068	0.066	0
5	0	0.072	0.075	0.077	0.078	0.078	0.078	0.077	0.075	0.072	0
6	0	0.08	0.083	0.085	0.086	0.086	0.086	0.085	0.083	0.08	0
7	0	0.088	0.091	0.094	0.095	0.095	0.095	0.094	0.091	0.088	...

// ChAS ni qatlamlarda yechish

$j := 1..n-1\ \ u^{\langle j+1\rangle} := lsolve(A, Bu^{\langle j-1\rangle} + 4u^{\langle j\rangle} + d^{\langle j\rangle})$

// Aniq va taqribiy yechim jadvalini , grafiklarni chiqarish

13.2.10-jadval

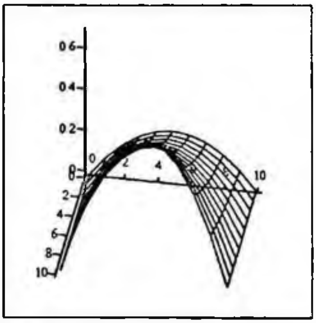
$ua^T =$

	0	1	2	3	4	5	6	7	8	9	10
0	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
1	0	0.099	0.177	0.232	0.265	0.276	0.265	0.232	0.177	0.099	0
2	0	0.11	0.195	0.256	0.293	0.305	0.293	0.256	0.195	0.11	0
3	0	0.121	0.216	0.283	0.324	0.337	0.324	0.283	0.216	0.121	0
4	0	0.134	0.239	0.313	0.358	0.373	0.358	0.313	0.239	0.134	0
5	0	0.148	0.264	0.346	0.396	0.412	0.396	0.346	0.264	0.148	0
6	0	0.164	0.292	0.383	0.437	0.456	0.437	0.383	0.292	0.164	0
7	0	0.181	0.322	0.423	0.483	0.503	0.483	0.423	0.322	0.181	0
8	0	0.2	0.356	0.467	0.534	0.556	0.534	0.467	0.356	0.2	...

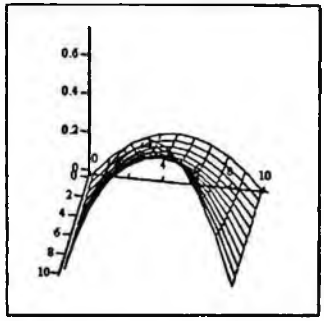
13.2.11-jadval

$u^T =$

	0	1	2	3	4	5	6	7	8	9	10
0	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
1	0	0.089	0.166	0.221	0.254	0.265	0.254	0.221	0.166	0.089	0
2	0	0.095	0.176	0.236	0.272	0.284	0.272	0.236	0.176	0.095	0
3	0	0.109	0.193	0.256	0.295	0.308	0.295	0.256	0.193	0.109	0
4	0	0.127	0.218	0.283	0.324	0.338	0.324	0.283	0.218	0.127	0
5	0	0.145	0.249	0.319	0.362	0.376	0.362	0.319	0.249	0.145	0
6	0	0.161	0.284	0.364	0.409	0.424	0.409	0.364	0.284	0.161	0
7	0	0.178	0.32	0.415	0.468	0.484	0.468	0.415	0.32	0.178	0
8	0	0.2	0.358	0.472	0.536	0.557	0.536	0.472	0.358	0.2	...



ua^T



u^T

13.2.2-rasm

Individual topshiriqlar.

GDTga chegara masala uchun oshkor va oshkormas sxemalar tuzilsin. Ular Mathcad, Maple dasturlarida ichki funksiyalar va algoritm tuzib echilsin, natijalarning yaqinligiga yerishilsin. Grafiklar chizilsin.

GDT uchun boshlang'ich-chegara masala echilsin:

$u_t - u_{xx} = f(x,t)$, $u(x,0) = g_1(x), u_t(x,0) = g_2(x)$, $u(0,t) = \mu_1(t), u(1,t) = \mu_2(t)$.

13.2.12-jadval

№	$u(x,t)$	$u(x,0)$	$u_t(x,0)$	$u(0,t)$	$u(1,t)$	$f(x,t)$
1-5	$x(1-x)\sin \pi t$	0	$x(1-x)$	0	0	$f(x,t) = u_{tt}(.) - u_{xx}(.)$
6-10	$x(1-x)\cos \pi t$	$x(1-x)$	0	0	0	$f(x,t) = u_{tt}(.) - u_{xx}(.)$
11-15	$x(1-x)\exp \pi t$	$x(1-x)$	$x(1-x)$	0	0	$f(x,t) = u_{tt}(.) - u_{xx}(.)$

16-20	$x(1-x)shnt$	0	$x(1-x)$	0	0	$f(x,t)=u_{tt}(.)-u_{xx}(.)$
21-25	$x(1-x)chnt$	$x(1-x)$	0	0	0	$f(x,t)=u_{tt}(.)-u_{xx}(.)$
26-30	$x(1-x)e^{-nt}$	0	$2x(1-x)$	0	0	$f(x,t)=u_{tt}(.)-u_{xx}(.)$
31-35	$\alpha x^3+\beta t^2$	αx^3	0	βt^2	$1+\beta t^2$	$f(x,t)=u_{tt}(.)-u_{xx}(.)$
36-40	$\alpha x^3+\beta t^3$	αx^3	0	βt^3	$1+\beta t^3$	$f(x,t)=u_{tt}(.)-u_{xx}(.)$

Nazariy savollar va topshiriqlar.

1.GDT uchun sof oshkormas sxema approksimasiyasi qanday?

2. CHegara shartning approksimasiyasi qanday?

3. GDT uchun Krank-Nikolson sxemasi approksimasiyasi qanday?

4.Oshkor sxema qanday turg'un. To'r qadamlari qanday tanlanadi?

5. Oshkormas sxemalar qanday turg'un. To'r qadamlari qanday tanlanadi?

6. Approksimasiya, turg'unlik, yaqinlashish nima. Fillipov teoremasi nima?

3. EDT uchun chekli ayirmali sxemalar

Asosiy nazariy tushunchalar

1.Elliptik tenglama uchun chegara masala:

$$Lu=u_{xx}+u_{yy}=f(x,y),(x,y)\in D,\alpha u+\beta\partial u/\partial n=g,(x,y)\in\Gamma .$$

2.CHAS tuzish va tekshirish.

$$L_h u_h=\frac{u_{i-1j}-2u_{ij}+u_{i+1j}}{h_1^2}+\frac{u_{ij-1}-2u_{ij}+u_{ij+1}}{h_2^2}=f_{ij},i=1..m-1;j=1..,n-1.$$
$$u_{ij}=g(x_i,y_j),i=0,m;j=0..n;j=0,n;i=0..m.$$

Approksimasiya: $R_h=L_hu_h-f_h=O(h_1^2+h_2^2)$ .Turg'unligi: $\|u_h\|_{U_h}\le C_1\|f_h\|_{F_h}+C_2\|g_h\|_{G_h}$.

3. CHAS ni yechish algoritmi. Lipman iterasiya usuli.

$$u_{ij}^{(k)}=\left(\frac{2}{h_1^2+h_2^2}\right)^{-1}\left[\frac{u_{i-1j}^{(k-1)}+u_{i+1j}^{(k-1)}}{h_1^2}+\frac{u_{ij-1}^{(k-1)}+u_{ij+1}^{(k-1)}}{h_2^2}-f_{ij}\right],i=1..m-1;j=1..n-1.$$
$$u_{ij}^{(k)}=g(x_i,y_j),i=0,m;j=0..n;j=0..n;i=0..m;j=0,n.$$

4. Matrisali progonka usuli. $u_{i+1}+Au_i+u_{i-1}=f_i,i=1..m-1,u_0=g_0,u_m=g_m$.

$$A=\begin{bmatrix}\lambda & \alpha & 0 & . & 0 & 0 & 0\\ \alpha & \lambda & \alpha & . & 0 & 0 & 0\\ . & . & . & . & . & . & .\\ 0 & 0 & 0 & . & \alpha & \lambda & \alpha\\ 0 & 0 & 0 & . & 0 & \alpha & \lambda\end{bmatrix},\lambda=-2(1+\alpha),u_i=\begin{bmatrix}u_{i1}\\ u_{i2}\\ .\\ u_{in-2}\\ u_{in-1}\end{bmatrix}$$

$$f_i=\begin{bmatrix}h^2f_{i1}-\alpha g_{i0}\\ h^2f_{i2}\\ .\\ h^2f_{in-2}\\ h^2f_{in-1}-\alpha g_{in}\end{bmatrix},g_0=\begin{bmatrix}g_{01}\\ g_{02}\\ .\\ g_{0n-2}\\ g_{0n-1}\end{bmatrix},g_m=\begin{bmatrix}g_{m1}\\ g_{m2}\\ .\\ g_{mn-2}\\ g_{mn-1}\end{bmatrix}$$

1. $R_1=0$ deb $R_i=[R_{ij}^{(i)}]_1^{n-1},i=1..m-1,$ matrisalarni quyidagi formulalar asosida hisoblaymiz: $R_{i+1}=-(A+R_i)^{-1},i=1..m-1.$

2. So'ng $s_1=g_0$ deb $s_i=[s_1^{(i)},s_2^{(i)},...,s_{n-1}^{(i)}]^T,i=1..m,$ vektorlarni ushbu formulalar bilan hisoblaymiz:

$$s_{i+1}=R_{i+1}(s_i-f_i),i=1..m-1.$$

3. $u_m=g_m$ deb teskari progonka asosida $u_{i-1}=R_iu_i+s_i,i=m..1$, (11) masalaning yechimini topamiz.

Lipman iterasiya usuli. Mathcadda algoritm.

Ushbu munosabatlarni qaraymiz:

$$u_{ij}=\frac{h_1^2h_2^2}{2(h_1^2+h_2^2)}[\frac{u_{i-1j}+u_{i+1j}}{h_1^2}+\frac{u_{ij-1}+u_{ij+1}}{h_2^2}-f_{ij}],i=1..m-1;j=1..n-1.$$
$$u_{ij}=g(x_i,y_j),i=0,m;j=0..n;i=0..m;j=0..n.$$

Bu tenglamalar sistemasiga Yakobi iterasiya usulini qo'llaymiz:

$$u_{ij}^{(k)}=\frac{h_1^2h_2^2}{2(h_1^2+h_2^2)}\left[\frac{u_{i-1j}^{(k-1)}+u_{i+1j}^{(k-1)}}{h_1^2}+\frac{u_{ij-1}^{(k-1)}+u_{ij+1}^{(k-1)}}{h_2^2}-f_{ij}\right],i=1..m-1;j=1..n-1.$$
$$u_{ij}^{(k)}=g(x_i,y_j),i=0,m;j=0..n;j=0..n;i=0..m;j=0,n.$$

Hisoblashlar ushbu tengsizliklar bajarilguncha bajariladi: $|u_{ij}^{(k)}-u_{ij}^{(k-1)}|$<eps. iterasiyalar yaqinlashuvchan. Mathcad matematik tizimida programma quyidagi ko'rinishga ega:

$n1:=10\ \ n2:=10\ \ i:=0..n1\ \ j:=0..n2\ \ h1:=L1/n1\ \ h2:=L2/n2\ \ x_i:=ih1\ \ y_j:=jh2$

$A:=1/h1^2\ \ B:=1/h2^2\ \ C:=(h1^2h2^2)/2/(h1^2+h2^2)$

$ua(x,y) := x^2 y^2 (x^2 + y^2) \qquad f(x,y) := 2(x^4 + 12x^2 y^2 + y^4) \quad ua_{i,j} := ua(x_i, y_j) \quad f_{i,j} := f(x_i, y_j)$

$u_{0,j} := 0 \;\; u_{i,0} := 0 \;\; u_{n1,j} := (jh2)^2 + (jh2)^4 \;\; u_{i,n2} := (ih1)^4 + (ih1)^2$

```
h(u,ε,n) := | v[n1,n2] ← 0
            | for k ∈ 0..n
            |   | u ← v  if k > 0
            |   | for i ∈ 1..n1 − 1
            |   |   for j ∈ 1..n2 − 1
            |   |     u[i,j] ← C·[A·(u[i+1,j] + u[i−1,j]) + B·(u[i,j+1] + u[i,j−1]) − f[i,j]]
            |   | e ← norme(v − u)
            |   | v ← u
            |   | break  if e < ε
            | u
```

$\varepsilon := 0.0001 \;\; n := 500 \;\; uu := h(u, \varepsilon, n)$

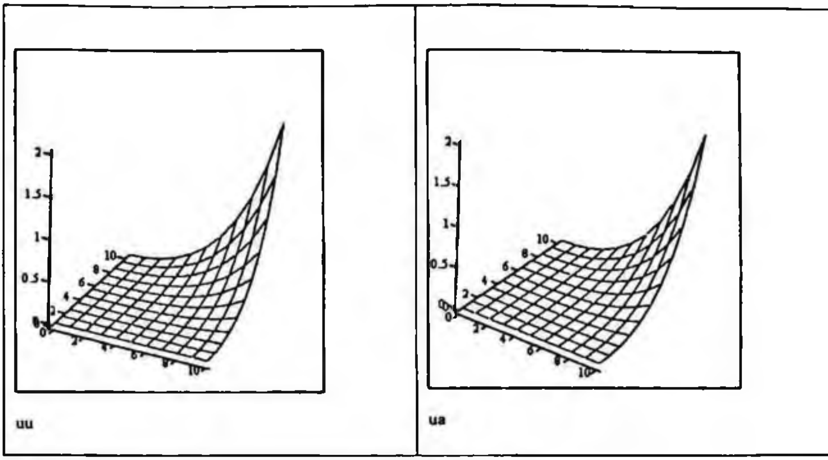
Bu yerda u a (x , y) = x (x f x) (, +) 2 (y aniq yechim va tenglamaning o'ng tomoni. $v=v_{n1,n2}$-nol matrisa va, *A*, *V*, S- CHAS ning koeffisientlari, norme- matrisaning evklid normasi. Yechimni ekranga chiqarish uchun ushbu komandani berish kerak:

h (u, 0.0001, 500)=:

13.3.1-jadval

0	0	0	0	0	0	0	0	0	0	0
0	0.00003	0.00008	0.00019	0.00041	0.00083	0.00154	0.00268	0.00438	0.0068	0.0101
0	0.00008	0.00025	0.00066	0.00154	0.00322	0.00613	0.01079	0.01778	0.02799	0.0416
0	0.00019	0.00088	0.00174	0.00397	0.0081	0.01509	0.0264	0.04253	0.06593	0.0981
0	0.00041	0.00154	0.00379	0.00867	0.01697	0.03058	0.0516	0.08249	0.12609	0.1856
0	0.00083	0.00322	0.0081	0.01697	0.03191	0.05563	0.09138	0.14304	0.21507	0.3125
0	0.00154	0.00613	0.01509	0.03058	0.05563	0.0941	0.15072	0.23109	0.34162	0.4896
0	0.00268	0.01079	0.02611	0.0516	0.09138	0.15072	0.23608	0.35505	0.51642	0.7301
0	0.00438	0.01778	0.04253	0.08249	0.14304	0.23109	0.35505	0.5249	0.75209	1.0496
0	0.0068	0.02779	0.06593	0.12609	0.21507	0.34162	0.51643	0.75209	1.06317	1.4661
0	0.0101	0.0416	0.0481	0.1856	0.3125	0.4896	0.7301	1.0496	1.4661	2

Grafiklarni chiqaramiz.



13.3.1-rasm

Xuddi shunday natijani *h(u, 0.0001,200)*= komandani berrib ham olish mumkin.

Matrisali progonka usuli.

Puasson tenglamasi .

$\Delta u = f(x,y), \phi 0(y) := ua(0,y)\ \ \phi m(y) := ua(1,y)\ , \psi 0(x) := ua(x,0)\ \ \psi n(x) := ua(x,1)$

uchun vektor –matrisali progonkani qaraymiz. Dastur quyidagicha yoziladi:

$u_{i+1} + Au_i + u_{i-1} = f_i, i = 1..n-1, u_i = [u_{i1},...,u_{i,m-1}]^T, i = 0..n, u_0 = \varphi_0, u_m = \varphi_m$ *// nomalumlar*

$A = [A_{i,j}], A_{i,i} = -2(1+\alpha), A_{i+1,i} = A_{i,i+1} = \alpha.$ *// vektor-matrisalar*

$f1_i := \left[h^2 f_{i1} - \alpha\psi_{i0}\ \ h^2 f_{i2}\ \ h^2 f_{i3}\ \ h^2 f_{i4}\ \ h^2 f_{i5}\ \ h^2 f_{i6}\ \ h^2 f_{i7}\ \ h^2 f_{i8}\ \ h^2 f_{i,m-1} - \alpha\psi_{m-1}\right]^T$ *//ozod had*

$\varphi_0 := \left[\varphi_{01}\ \ \varphi_{02}\ \ \varphi_{03}\ \ \varphi_{04}\ \ \varphi_{05}\ \ \varphi_{06}\ \ \varphi_{07}\ \ \varphi_{08}\ \ \varphi_{0,n-1}\ \right]^T$, *//chegaradagi qiymatlar*

$\varphi_m := \left[\varphi_{m1}\ \ \varphi_{m2}\ \ \varphi_{m3}\ \ \varphi_{m4}\ \ \varphi_{m5}\ \ \varphi_{m6}\ \ \varphi_{m7}\ \ \varphi_{m8}\ \ \varphi_{m,n-1}\ \right]^T$ *// chegaradagi qiymatlar*

Vektor –matrisali progonkani quyidagi ko'rinishga ega[23]:

1) $R_1 = [0_{ij}]_{i,j=1}^{n-1}, k = 1..m-1\ R_{k+1} = -(A + R_k)^{-1}$

2) $s1 := 0 \quad k = 1..m-1 \quad s_{k+1} := R_{k+1}(s_k - f_k)$

3) $u_m := \varphi_m \quad k := m, m-1..1\ \ u_{k-1} := Ru_k + s_k$

Ushbu masalani qaraymiz: $u_{xx} + u_{yy} = -4$.

$ua(x,y) := x^2 + y^2\ \ u(0,y) = y^2\ \ u(1,y) = 1 + y^2\ \ u(x,0) = x^2\ \ u(x,1) = 1 + x^2$

$\phi 0(y) := ua(0,y)$ $\phi m(y) := ua(1,y)$ $\psi 0(x) := ua(x,0)$ $\psi n(x) := ua(x,1)$ $ua(x,y) := x^2 + y^2$

$m := 10$ $n := 10$ $h1 := 1/m$ $h2 := 1/n$ $\alpha := (h1/h2)^2$

$i := 0..m$ $j := 0..n$ $x_i := i * h1$ $y_j := j * h2$

$j := 1..n-1$ $a_{j,j+1} := \alpha$ $a_{j+1,j} := \alpha$ $a_{j,j} := -2(1+\alpha)$

$A := submatrix(a,1,9,1,9)$

$i := 1..n-1$ $f_{i,j} := f(x_i, y_j)$ $\psi 0_i := \psi 0(x_i)$ $\psi n_i := \psi n(x_i)$

$f1_i := \begin{bmatrix} h^2 f_{i1} - \alpha\psi_{i0} & h^2 f_{i2} & h^2 f_{i3} & h^2 f_{i4} & h^2 f_{i5} & h^2 f_{i6} & h^2 f_{i7} & h^2 f_{i8} & h^2 f_{m-1} - \alpha\psi_{m-1} \end{bmatrix}^T$

$\varphi_0 := \begin{bmatrix} \varphi_{01} & \varphi_{02} & \varphi_{03} & \varphi_{04} & \varphi_{05} & \varphi_{06} & \varphi_{07} & \varphi_{08} & \varphi_{0n-1} \end{bmatrix}^T$

$\varphi_m := \begin{bmatrix} \varphi_{m1} & \varphi_{m2} & \varphi_{m3} & \varphi_{m4} & \varphi_{m5} & \varphi_{m6} & \varphi_{m7} & \varphi_{m8} & \varphi_{mn-1} \end{bmatrix}^T$

$n := 9$ $R1 := identity(9)$ $R1_{i,j} := 0$ $R := submatrix(R1,1,9,1,9)$

$s1 := 0$ $u_m := \varphi_m$

$k := 1..m-1$ $Q := -(A+R)^{-1}$ $s_{k+1} := Q(s_k - f_k)$ $R := Q$

$p := m, m-1..1$ $u_{k-1} := Ru_k + s_k$ *// barcha ustundagi qiymatlar*

$U := augment(u_0, u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10})$ *// yechim matrisasini tuzish*

// Javobni birdaniga chiqarish:

13.3.2-jadval

U =

	0	1	2	3	4	5	6	7	8	9	10
0	0.017	0.043	0.068	0.1	0.143	0.198	0.269	0.361	0.489	0.682	1.01
1	0.025	0.054	0.076	0.097	0.123	0.158	0.207	0.281	0.403	0.621	1.04
2	0.029	0.059	0.078	0.094	0.113	0.138	0.178	0.247	0.372	0.613	1.09
3	0.033	0.063	0.082	0.096	0.112	0.135	0.172	0.241	0.375	0.637	1.16
4	0.039	0.072	0.093	0.108	0.124	0.147	0.186	0.258	0.4	0.683	1.25
5	0.052	0.092	0.119	0.139	0.158	0.184	0.225	0.302	0.452	0.753	1.36
6	0.076	0.138	0.18	0.209	0.235	0.265	0.312	0.394	0.549	0.858	1.49
7	0.115	0.239	0.311	0.356	0.393	0.434	0.49	0.579	0.734	1.031	1.64
8	0.146	0.467	0.579	0.638	0.691	0.751	0.826	0.929	1.079	1.33	1.81

Ichki funksiyalar yordamida yechish.

Misol 1. $u_{xx} + u_{yy} = -2(x - x^2 + y - y^2)$ tenglama, aniq yechim

$ua = (x - x^2)(y - y^2), 0 \le x \le 1, 0 \le y \le 1$.

$u(x,0) = 0, u(x,1) = 0, u(0,y) = 0, u(1,y) = 0$.

Misolni $relax(a,b,c,d,e,\rho,u,r)$ ichki funksiya bilan echamiz, parametrlar EDT ning chekli ayirmali ko'rinishidan olinadi:

$$au_{i+1,j}+bu_{i-1,j}+cu_{i,j+1}+du_{i,j-1}+eu_{i,j}=\rho_{i,j},$$

$$a=b=c=d, e=-4a, a_{i,j}=1, i:=1..m, j:=1..m, 0<r<1.$$

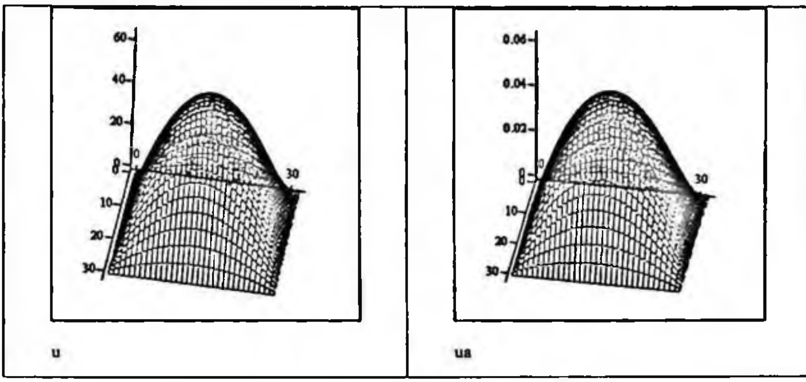
r –relaksasiya koeffisenti, usulning yaqinlashishini ta'minlaydi.

$n:=32\quad L1=1\ L2:=1\ h1:=1/n1\ \ h2:=1/n2\ \ ua(x,y):=x*y*(1-x)*(1-y)$

$R:=32\ i:=0..n1\ \ j:=0..n2\ x_i:=i*h1\ \ y_j:=j*h2\quad f(x,y):=-2(x+y-x^2-y^2)$

$a_{i,j}:=1\ b:=a\ c:=a\ d:=a\ e:=-4*a\ f_{i,j}:=f(x_i,y_j)\ r:=1-2*\pi/R$

$u_{i,0}:=0\ u_{0,j}:=0\ u_{n1,j}:=0\ u_{i,n2}:=0\quad F:=relax(a,b,c,d,e,f,u,r)\quad ua_{i,j}:=ua(x_i,y_j)$



13.3.2-rasm

Individual topshiriqlar.

EDTga chegara masala uchun chekli ayirmali sxema tuzilsin va ular Mathcad, Maple dasturlarida ichki funksiyalar va algoritm tuzib echilsin, natijalarning yaqinligiga yerishilsin. Grafiklar chizilsin.

$$\Delta u=u_{xx}+u_{yy}=f(x,y),\ u(0,y)=g_1(y), u(1,y)=g_2(y), u(x,0)=g_3(x), u(x,1)=g_4(x)$$

13.3.2-jadval

№	$u(x,y)$	$u(0,y)$	$u(1,y)$	$u(x,0)$	$u(x,1)$	$f=u_{xx}+u_{yy}$
1-5	$(x-x^2)\sin(\pi y/2)$	0	0	0	$x-x^2$	
6-10	$(x-x^2)\cos(\pi y/2)$	0	0	$x-x^2$	0	
11-15	$(y-y^2)\sin(x\pi/2)$	0	0 $y-y^2$	0	0	
16-20	$(y-y^2)\cos(x\pi/2)$	$y-y^2$	0	0	0	
21-25	$(x-x^2)sh(\pi y/2)$	0	0	0	$(x-x^2)sh(\pi/2)$	
26-30	$(x-x^2)ch(\pi y/2)$	0	0	$x-x^2$	$(x-x^2)ch(\pi/2)$	
31-35	$\alpha x^3+\beta y^3+xy$	βy^3	$\alpha+\beta y^3+y$	αx^3	$\alpha x^3+\beta+x$	$6\alpha x+6\beta y$
36-40	$\alpha x^4+\beta y^3+xy$	βy^3	$\alpha+\beta y^3+y$	αx^4	$\alpha x^4+\beta+x$	$12\alpha x+6\beta y$

Nazariy savol va topshiriqlar.

1. EDT nima?
2. EDT uchun boshlang'ich shart nima?
4. EDT uchun oshkor sxema approksimasiyasi qanday?
5. CHegara shartning approksimasiyasi qanday?
6. Approksimasiya, turg'unlik, yaqinlashish nima.
7. EDT uchun Lipman iterasiyasi qanday?
8. EDT uchun ayirmali sxemani yechishning yana qanday usullarini bilasiz ?

4. Integral tenglamalarni taqribiy yechish

Qisqacha nazariy malumot.

2-tur Voltyer va Fredgolm integral tenglamalari quyidagi ko'rinishda beriladi:

$$Lu \equiv u(x) = \lambda\int_a^x K(x,y)u(y)dy + f(x), a \le x \le b, \qquad (1)$$

$$Lu \equiv u(x) = \lambda\int_a^b K(x,y)u(y)dy + f(x), a \le x \le b. \qquad (2)$$

A. IT ni yechish uchun CHAS yoki kvadraturlar usuli.

G'oya quyidagidan iborat. [a,b] kesmada to'r-nuqtalar to'plami kiritamiz:

$\Delta_n = \{x_i = a + (i-1)h, i = 1..n, h = (b-a)/(n-1)\}$.

Aniq yechim $\bar{u} = \bar{u}(x) = u(x)$ va taqribiy yechim $u = u(x)$ ning to'rdagi qiymatlarini quyidagicha belgilaymiz: $\bar{u}_i = u_i = \bar{u}(x_i)$, $u_i = u(x_i)$, $i = 1..n$. Ushbu tugun nuqtalari $\{x_i\}$ va koeffisientlari $\{c_i\}$ kvadratura formulasini qaraymiz:

$$\int_a^b f(x)dx = \sum_{j=1}^{n} c_j f(x_j) + R_n(f), R_n(f) = O(h^r). \tag{3}$$

Masalan, trapesiya va Simpson kvadratura formulalari uchun, agar $h = (b-a)/(n-1), n = 2m+1, x_i = a + (i-1)h$, desak u holda quyidagilar o'rinli:

1) $A_1 := h/2, A_i := h/2, i := 2..n-1, A_n := h$, $R_n = -(b-a)f^{(2)}(\xi)h^2/12$,

2) $B_1 = h/3, B_n = h/3; i = 1..m, B_{2i} = 4h/3; i = 2..m, B_{2i-1} = 2h/3, R_n = -(b-a)f^{(4)}(\xi)h^4/180$.

(2) tenglamada $x = x_i$ deb, integralni integral yig'indi bilan almashtirib, quyidagi tenglikni hosil qilamiz:

$$u(x_i) - \lambda\sum_{j=1}^{n} c_j K(x_i, x_j)u(x_j) = f(x_i) + R_n(\lambda K(x_i, x)u(x)).$$

Qoldiq hadni tashlab uborib, quyidagi belgilashlarni kiritamiz:

$$u_i = u(x_i) \approx \bar{u}(x_i) = \bar{u}_i, K_{ij} = K(x_i, x_j), f_i = f(x_i), i = 1..n.$$

U holda taqribiy yechimning qiymatlarini (koeffisienlarini) topish uchun quyidagi CHATS ni olamiz:

$$u_i - \lambda\sum_{j=1}^{n} c_j K_{ij} u_j = f_i, i = 1..n. \tag{4}$$

Agar $Eu = u, K = [K_{ij}], u = [u_i], f = [f_i], c = [c_i]$, belgilashlarni kiritsak (4) CHATS ni quyidagicha yozish mumkin: $Mu = f, M = E - \lambda cK$. Agar $\det(M) \neq 0$ bo'lsa Mathcadda yechim ushbu formula $u = M^{-1}f$ yoki $u = lsolve(M, f)$ bilan beriladi. Xuddi shu kabi, Voltyer ITning taqribiy yechimining qiymatlarini (koeffisienlarini) topish uchun ushbu CHATS ni olamiz:

$$u_i - \lambda\sum_{j=1}^{i} c_i K_{ij} u_j = f_i, i = 1..n. \tag{5}$$

Taqribiy yechimning o'zini ushbu interpolyasiya formulasi ko'rinishda yozish mumkin:

$$u_n(x)=f(x)+\lambda_i\sum_{j=1}^{n}c_jK(x,x_j)u_j\,. \qquad (6)$$

Agar $1-\lambda c_iK(x_i,x_i)\neq 0$, bo'lsa, u holda (5) CHATS quyidagicha echiladi:

$$u_1=(1-\lambda c_1K_{11})^{-1}f_1, u_i=(1-\lambda c_iK_{ii})^{-1}(f+\lambda\sum_{j=1}^{i-1}c_iK_{ij}u_j), i\geq 2\,. \qquad (7)$$

Volterra IT si uchun yana bir boshqa formula chiqaramiz. (5) tenglikni chiqarishda integralni $[a,x_i]$ kesmada approksimasiya qilishni talab qilamiz, kvadratura formula har bir $[a,x_i]$ kesmada to'liq qo'llanilsin va $c_1+..+c_i=x_i-a$ tenglik bajarilsin. 2 tur Voltyer IT ni qaraymiz:

$$Lu\equiv u(x)=\lambda\int_a^x K(x,y)u(y)dy+f(x), a\leq x\leq b\,.$$

Bu yerda integralda ketma-ket $x=x_i, i=1..n$ deb olamiz:

$$Lu(x_i)\equiv u(x_i)=\lambda\int_a^{x_i} K(x_i,y)u(y)dy+f(x_i), i=1..n\,.$$

Bu yerdan darrov olamiz: $u_1=f_1$. So'ng, integralga $[a,x_i]$ kesmada trapesiya formulasini qo'llaymiz:

$$u_i=\lambda\sum_{j=1}^{i}c_jK_{i,j}u_j+f_i, i=2..n, c_1=c_i=h/2, c_2=..=c_{i-1}=h\,.$$

Natijada ushbu formulalar ketma-ketligini topamiz:

$$u_1=f_1,\;\; u_i=(f_i+\lambda\sum_{j=2}^{i-1}c_jK_{i,j}u_j)/(1-\lambda hK_{i,i}/2), i=2..n\,. \qquad (8)$$

V. Uzluksiz iterasiya usulining g'oyasi quyidagidan iborat: $\{u^{(m)}(x)\}$ ketma-ketlik quyidagicha quriladi:

$$u^{(0)}(x)=0, u^{(m+1)}(x)=f(x)+\lambda\int_a^b K(x,y)u^{(m)}(y)dy\,, \qquad (9)$$

$$u^{(0)}(x)=0, u^{(m+1)}(x)=f(x)+\lambda\int_a^x K(x,y)u^{(m)}(y)dy\,. \qquad (10)$$

Agar integrallar qiyinchilik bilan hisoblanadigan bo'lsa, ular quyidagicha taqribiy hisoblanishi mumkin va iterasiya usulining diskret varianti kelib chiqadi:

$$u_i^{(0)}=0, u_i^{(m+1)}=f_i+\lambda\sum_{j=1}^{n}c_jK_{i,j}u_i^{(m)}, m=0,1,..;i=1..n, \qquad (9')$$

$$u_i^{(0)}=0, u_i^{(m+1)}=f_i+\lambda\sum_{j=1}^{i}c_jK_{i,j}u_i^{(m)}, m=0,1,..;i=1..n. \qquad (10')$$

Hisoblashlar berilgan aniqlik o'rnatilguncha olib boriladi:

$$\left|u_i^{(m+1)}-u_i^{(m)}\right|<\varepsilon, \forall i=1..n.$$

(9), (10) IT yerasion jarayonlarning yaqinlashishi ularni berruvchi op yeratorlarning qisqartirib aks ettitrish xususiyatlariga bog'liq, jumladan o'ng tomonlardagi matrisalarning xossalaridan bog'liq.

C. Yadroni aynigan yadro bilan almashtirish.

Agar yadroning o'zgaruvchilari ajraladigan va chekli yig'indi bo'lsa, u aynigan yadro deyiladi:

$$K(x,y)=K_n(x,y)=\sum_{j=1}^{n}\alpha_j(x)\beta_j(y), \qquad (11)$$

(11) ni (2) ga qo'yib, topamiz:

$$u(x)=u_n(x)=f(x)+\sum_{j=1}^{n}c_j\alpha_j(x), \qquad (12)$$

bu yerda

$$c_i=\lambda\int_a^b\beta_i(y)u(y)dy. \qquad (13)$$

(12) ni (13) ga qo'yib, c_i koeffisientlarni topish uchun, ushbu CHATS ni topamiz:

$$c_i-\lambda\sum_{j=1}^{n}K_{ij}c_j=f_i, K_{ij}=\int_a^b\alpha_j(x)\beta_i(x)dy,\ f_i=\int_a^b f(x)\beta_i(x)dx. \qquad (14)$$

Ixtiyoriy yadroni aynigan yadroga keltirish uchun yadroni Teylor formulasiga yoyish, interpolyasiya formulalari, Galyorkin usullari, momentlar usullaridan foydalanish mumkin. Masalan, Galyorkin usulida tafovut $R_n(x)=Lu_n(x)-f(x)$ bazis funksiyalarga ortogonal bo'lishi talab etiladi: $R_n(x)\perp\varphi_i(x)=0, i=1..n$. Bu bizga $Mc=f$ CHATS ni beradi:

$$M_{i,j}=\alpha_{i,j}-\lambda\beta_{i,j}, \alpha_{i,j}=\int_a^b\varphi(i,x)\varphi(j,x)dx,$$

$$\beta_{i,j}=\int_a^b(\int_a^b K(x,y)\varphi(j,y)dy)\varphi(i,x)dx,\ f_i=\int_a^b(\int_a^b K(x,y)f(y)dy)\varphi(i,x)dx,\ i,j=1..n.$$

D. Galyorkin, kollokasiya i eng kichik kvadratlar usuli.

Taqribiy yechimni ushbu formula ko'rinishda izlaymiz:

$$u_n(x)=\sum_{j=1}^n c_j\varphi_j(x)\quad (u_n(x)=f(x)+\sum_{j=1}^n c_j\varphi_j(x)),\qquad (15)$$

bu yerda $\{\varphi_i(x)\}$ funksiyalar $[a\ b$ kesmada Chiziqli yerkli to'liq funksiyalar sistemasi. Ayirma

$$R_n(x)=R_n(x,c_1,\dots,c_n)=Lu_n(x)-f(x)\qquad (16)$$

taqribiy $u_n(x)$ ning tafovuti deyiladi, u taqribiy yechim aniq tenglamani qanoatlantirish darajasini belgilaydi. Kollokasiya, Galyorkin, eng kichik kvadratlar usulida $c_1,\dots,c_n$ nomalumlar quyidagi shartlar asosida aniqlanadi:

1) kollokasiya usulida $Lu_n(x_i)-f(x_i)=0, i=1..n$;

2) Galyorkin usulida $R_n(x)\perp\varphi_i(x)=0, i=1..n$;

3) eng kichik kvadratlar usulida $c_i, i=1..n$ lar bo'yicha $\int_a^b[R_n(x)]^2dx\to\min$.

Bu talablar bizni ushbu CHATS ga olib keladi:

$$Mc=f,\qquad (17)$$

bu yerda $Mc=f$ sistema elementlari ushbu funksiya

$$\psi_j(x)=L\varphi_j(x)=\varphi_j(x)-\lambda\int_a^b K(x,y)\varphi_j(y)dy$$

asosida quyidagicha aniqlanadi va umumiy nazariya bilan ustma-ust tushadi:

$$1)\ M_{ij}=\psi_j(x_i), f_i=f(x_i), i=1..n.$$

$$2)\ M_{ij}=\int_a^b\varphi_i(x)\psi_j(x)dx, f_i=\int_a^b f(y)\varphi_i(y)dy, i=1..n,\qquad (18)$$

$$3)\ M_{ij}=\int_a^b\psi_i(x)\psi_j(x)dx, f_i=\int_a^b f(y)\psi_i(y)dy, i=1..n.$$

VOLTYER INTEGRAL TENGLAMALARINI TAQRIBIY YECHISH

A. Kvadratura formulalarini qo'llash.

Misol 1. Trapesiya formulasi asosida Voltyer IT echilsin:

$K(x,y):=e^{-(x-y)}\sin(x-y)$, $f(x):=e^{-x}$, $ut(x):=e^{-x}(1+0.5x^2)$., a:=0, b:=1. (19)

Mathcad oynasida quyidagi komandalarni teramiz va javobni olamiz:

Vol'ter IT 2 tur Trapesiya kvadratura formulasi $ORIGIN:=1$

$K(x,y):=e^{-(x-y)}\sin(x-y)$ $f(x):=e^{-x}$ $ut(x):=e^{-x}(1+0.5x^2)$

$a:=0$ $b:=1$ $n:=101$ $h:=(b-a)/(n-1)$

$i:=1..n$ $x_i:=a+(i-1)h$ $f_i:=f(x_i)$ $ut_i:=ut(x_i)$ $j:=1..n$ $K_{i,j}:=K(x_i,x_j)$

$A_1:=h/2$ $A_n:=h/2$ $k:=2..n-1$ $A_k:=h$

$$u_1:=f_1 \; k:=2..n \; u_k:=(1-hK_{k,k}/2)^{-1}(f_k+\sum_{j=1}^{n} if(j\ge k,0,A_jK_{k,j}u_j))$$

$k:=1..n$ $v_k:=u_k$ $vt_k:=ut_k$

13.4.1-jadval

$v^T=$

	1	2	3	4	5	6	7	8	9	10
1	1	0.99	0.98	0.97	0.961	0.951	0.942	0.932	0.923	0.914

13.4.2-jadval

$vt^T=$

	1	2	3	4	5	6	7	8	9	10
1	1	0.99	0.98	0.971	0.962	0.952	0.943	0.935	0.926	0.918

Misol 2. Trapesiya-Simpson usuli bilan Voltyer IT echilsin:

$K(x,y):=e^{-(x-y)}\sin(x-y)$, $f(x):=e^{-x}$, $ut(x):=e^{-x}(1+0.5x^2)$, a:=0, b:=1.

Indeks $i:=2..n$ ning toq yoki juftligiga qarab, $[a,x_i]$ kesmada navbati bilan trapesiya yoki Simpsona kvadratura formulasi qo'llaniladi.

Mathcad oynasida ushbu komandalarni teramiz va natijalar olamiz:

Вольтер ИТ 2 тур Трапеция + Симпсон КФ $ORIGIN:=1$

$K(x,y):=e^{-(x-y)}\sin(x-y)$ $f(x):=e^{-x}$ $ut(x):=e^{-x}(1+0.5x^2)$

$a:=0$ $b:=1$ $m:=40$ $n:=2m+1$ $h:=(b-a)/(n-1)$

$i:=1..n$ $x_i:=a+(i-1)h$ $f_i:=f(x_i)$ $ut_i:=ut(x_i)$ $j:=1..n$ $K_{i,j}:=K(x_i,x_j)$

$A_1:=h/2$ $A_n:=h/2$ $k:=2..n-1$ $A_k:=h$

$B_1:=h/3$ $B_n:=h/3$ $k:=1..m$ $B_{2k}:=4h/3$ $k:=2..m$ $B_{2k-1}:=2h/3$

$$T_1:=f_1 \; k:=2,4..2m \; T_k:=(1-hK_{k,k}/2)^{-1}(f_k+\sum_{j=1}^{k-1}A_jK_{k,j}T_j)$$

13.4.3-jadval

$T^T =$

	1	2	3	4	5	6	7	8	9	10	11
1	1	0.988	0	0.964	0	0.941	0	0.918	0	0.897	0

$$S_1 := f_1 \quad k := 3,5..2m+1 \quad S_k := (1 - hK_{k,k}/3)^{-1}(f_k + \sum_{j=1}^{k-1} B_j K_{k,j} S_j)$$

13.4.4-jadval

$S^T =$

	1	2	3	4	5	6	7	8	9	10	11
1	1	0	0.975	0	0.952	0	0.929	0	0.906	0	0.885

$k := 1..n \quad u_k := if(\mathrm{mod}(k,2) = 0, T_k, S_k)$

13.4.5-jadval

$u^T =$

	1	2	3	4	5	6	7	8	9	10	11
1	1	0.988	0.975	0.964	0.952	0.941	0.929	0.918	0.906	0.897	0.885

13.4.6-jadval

$u^T =$

	1	2	3	4	5	6	7	8	9	10	11
1	1	0.988	0.976	0.964	0.952	0.941	0.93	0.92	0.909	0.899	0.889

Bu yerda o'zgaruvchilar T^T, S^T ning qiymatlari nazorat uchun chiqarilgan.

V. Diskret iterasiya usuli. IT uchun trapesiya formulasi asosida iterasiyalar ketma-ketligini quramiz:

a) trapesiya usuli:

$$u_i^{(m+1)} = f_i + \frac{h}{2}[K_{i1}u_1^{(m)} + 2\sum_{j=2}^{i-1} K_{ij}u_j^{(m)} + K_{ii}u_i^{(m)}], i = 1,...,n. \quad (20)$$

b) Simpson usuli:

$$u_i^{(m+1)} = f_i + \frac{h}{3}[K_{i1}u_i^{(m)} + 2\sum_{j=2}^{i/2}[2K_{i,2j-1}u_{2j-1}^{(m)} + K_{i,2j}u_{2j}^{(m)}] + K_{i,i}u_i^{(m)}]. \quad (21)$$

Misol 3. Trapesiya usuli bilan Voltyer IT echilsin.

$$K(x,y) = x^2 + xy,\ \lambda = 0.1,\ f(x) = x^3 - \lambda(0.2x + 0.25x^2), a = 0, b = 1. \quad (22)$$

Mathcad oynasida ushbu komandalarni teramiz va natijalar olamiz:

Vol'ter 2tur Iterasiya usuli Trapesiya KF ORIGIN $:= 1$

$K(x,y) := x^2 + xy \quad \lambda := 0.1 \quad f(x) := x^3 - \lambda(0.2x + 0.25x^2)$

$a := 0 \quad b := 1 \quad m := 5 \quad n := 2m+1 \quad h := (b-a)/(n-1)$

$i := 1..n \quad x_i := a + (i-1)h \quad f_i := f(x_i) \quad j := 1..n \quad K_{i,j} := K(x_i, x_j)$

$A_1 := h/2 \ A_n := h/2 \ k := 2..n-1 \ A_k := h$

$u_1 := f_1 \ k := 2..n \ u_k := (1 - h\lambda K_{k,k}/2)^{-1}(f_k + \lambda \sum_{j=1}^{n} if(j \ge k, 0, A_j K_{k,j} u_j))$ // *trapesiy us.*

$u1_{1,j} := f_1 \ s := 1..m-1 \ i := 2..n \ u1_{s+1,i} := (f_i + \lambda \sum_{j=1}^{n} if(j \ge i, 0, A_j K_{i,j} u_j))$ // *ityerasiy us.*

13.4.7-jadval

$u^T =$

	1	2	3	4	5	6	7	8	9	10	11
1	0	-0.001	0.003	0.019	0.052	0.11	0.198	0.323	0.494	0.72	0.999

13.4.8-jadval

$u1 =$

	1	2	3	4	5	6	7	8	9	10	11
1	0	0	0	0	0	0	0	0	0	0	0
2	0	-0.001	0.003	0.019	0.052	0.109	0.196	0.32	0.488	0.708	0.989
3	0	-0.001	0.003	0.019	0.052	0.109	0.196	0.32	0.488	0.708	0.989
4	0	-0.001	0.003	0.019	0.052	0.109	0.196	0.32	0.488	0.708	0.989
5	0	-0.001	0.003	0.019	0.052	0.109	0.196	0.32	0.488	0.708	0.989

Misol 4. 1 tur Voltyer IT echilsin:

$K(x,y) := 2 + x^2 - y^2 \quad f(x) := x^2 \quad ut(x) := x\exp(-x^2/2)$.

Mathcad oynasida ushbu komandalarni teramiz va natijalar olamiz:

Vol'ter 1 tur IT Trapesiya KF ORIGIN $:= 1$

$K(x,y) := 2 + x^2 - y^2 \quad f(x) := x^2 \quad ut(x) := x\exp(-x^2/2)$

$a := 0 \ b := 3.5 \ m := 25 \ n := 2m+1 \ h := (b-a)/(n-1)$

$i := 1..n \ x_i := a + (i-1)h \ f_i := f(x_i) \ j := 1..n \ K_{i,j} := K(x_i, x_j)$

$A_1 := h/2 \ A_n := h/2 \ k := 2..n-1 \ A_k := h \ f1(x) := \frac{df(x)}{dx} \ u_1 := f1(a)$

$$u_1 := 0 \ k := 2..n \ u_k := (A_k K_{k,k})^{-1}(f_k - \lambda \sum_{j=1}^{k-1} A_j K_{k,j} u_j)$$

13.4.9-jadval

$u^T =$

	1	2	3	4	5	6	7	8	9	10	11
1	0	0.035	0.105	0.173	0.24	0.303	0.362	0.416	0.465	0.508	0.544

13.4.10-jadval

$ut^T =$

	1	2	3	4	5	6	7	8	9	10	11
1	0	0.07	0.139	0.205	0.269	0.329	0.385	0.435	0.479	0.517	0.548

4.3. FREDGOLM INTEGRAL TENGLAMALARINI TAQRIBIY YECHISH

A. Kvadratura formulasi.

Misol 1. Fredgolm 2-tur IT echilsin.

Mathcad oynasida ushbu komandalarni teramiz va natijalar olamiz:

Fredgolm 2tur Trapesiya KF $ORIGIN := 1$

$K(x,y) := xy \quad f(x) := 2x \quad ut(x) := 3x$

$a := 0 \quad b := 1 \quad n := 11 \quad h := (b-a)/(n-1)$

$i := 1..n \quad x_i := a + (i-1)h \quad f_i := f(x_i) \quad ut_i := ut(x_i) \quad j := 1..n \quad K_{i,j} := K(x_i, x_j)$

$A_1 := h/2 \quad A_n := h/2 \quad k := 2..n-1 \quad A_k := h$

$u_1 := f_1 \quad k := 2..n \quad u_k := (1 - hK_{k,k}/3)^{-1}(f_k + \sum_{j=1}^{n} if(j \ge k, 0, A_j K_{k,j} u_j))$

$i := 1..n \quad j := 1..n \quad M_{i,j} := if(j = i, 1 - A_i K_{i,i}, -A_j K_{i,j}) \quad u := lsolve(M, f)$

13.4.11-jadval

$u^T =$

	1	2	3	4	5	6	7	8	9	10	11
1	0	0.301	0.602	0.902	1.203	1.504	1.805	2.105	2.406	2.707	3.008

13.4.12-jadval

$ut^T =$

	1	2	3	4	5	6	7	8	9	10	11
1	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3

Misol 2. Fredgolm 2-tur IT Simpson usuli bilan echilsin.

Mathcad oynasida ushbu komandalarni yozamiz va natijalarni olamiz:

Fredgol'm 2tur IT Simpson KF $ORIGIN := 1$

$K(x,y) := \sin(x+2y) \quad f(x) := x \quad ut(x) := x - \pi\cos(x)$

$a := -\pi \quad b := \pi \quad m := 5 \quad n := 2m+1 \quad h := (b-a)/(n-1)$

$i := 1..n \quad x_i := a + (i-1)h \quad f_i := f(x_i) \quad ut_i := ut(x_i) \quad j := 1..n \quad K_{i,j} := K(x_i, x_j)$

$A_1 := h/3 \quad A_n := h/3 \quad k := 2,4..2m \quad A_k := 4h/3 \quad k := 3,5..2m-1 \quad A_k := 2h/3$

$i := 1..n \quad j := 1..n \quad M_{i,j} := if(j = i, 1 - A_i K_{i,i}, -A_j K_{i,j}) \quad u := lsolve(M, f)$

13.4.13-jadval

$u^T =$

	2	3	4	5	6	7	8	9	10
1	0.071	-0.898	-2.244	-3.213	-3.195	-1.956	0.269	2.872	...

13.4.14-jadval

$ut^T =$

	1	2	3	4	5	6	7	8	9	10
1	0	0.028	-0.914	-2.227	-3.17	-3.142	-1.913	0.286	2.856	...

V. Diskret iterasiya usuli.

Misol 3. Fredgolm 2-tur IT iterasiya usuli bilan echilsin.

Mathcad oynasida ushbu komandalarni yozamiz va natijalarni olamiz:

Fredgol'm 2tur Iterasiya usuli + Trapesiya KF $ORIGIN := 1$

$K(x,y) := 1 \;\; \lambda := 0.5 \;\; f(x) := \sin(\pi x) \;\; ut(x) := \sin(\pi x) + 2/\pi$

$a := 0 \;\; b := 1 \;\; m := 5 \;\; n := 2m+1 \quad h := (b-a)/(n-1)$

$i := 1..n \;\; x_i := a + (i-1)h \;\; f_i := f(x_i) \;\; ut_i := ut(x_i) \;\; j := 1..n \;\; K_{i,j} := K(x_i, x_j)$

$A_1 := h/2 \;\; A_n := h/2 \;\; k := 2..n-1 \;\; A_k := h$

$$u1_{1,i} := f_i \;\; s := 1..m-1 \;\; i := 2..n \;\; u1_{s+1,i} := f_i + \lambda \sum_{j=1}^{n} A_j K_{i,j} u_{s,j})$$

13.4.15-jadval

	1	2	3	4	5	6	7	8	9	10
6	0.612	0.921	1.199	1.421	1.563	1.612	1.563	1.421	1.199	0.921
7	0.622	0.931	1.209	1.431	1.573	1.622	1.573	1.431	1.209	0.931
8	0.622	0.935	1.214	1.435	1.577	1.626	1.577	1.435	1.214	0.935
9	0.629	0.938	1.217	1.438	1.58	1.629	1.58	1.438	1.217	0.938
10	0.63	0.939	1.218	1.439	1.581	1.63	1.581	1.439	1.218	...

13.4.16-jadval

$ut^T =$

	1	2	3	4	5	6	7	8	9	10
1	0.637	0.946	1.224	1.446	1.588	1.637	1.588	1.446	1.224	...

S. YAdroni aynigan yadro bilan almashtirish.

Misol 4. Faraz qilaylik IT berilgan bo'lsin:

$$u(x) + \int_0^{0.5} x sh(xy) dy = f(x), f(x) = 2 - ch(x/2) \cdot \tag{23}$$

Bazis funksiyalarni $\varphi_i(x) = x^{i-1}$, taqribiy yechimni ushbu ko'rinishda olamiz:

$$u(x) = u_n(x) = f(x) + \sum_{j=1}^{n} c_j \varphi_j(x).$$

Mathcad oynasida ushbu komandalarni yozamiz va natijalarni olamiz:

Fredgol'm 2tur IT Momentlar usuli + Aynigan yadro $ORIGIN := 1$

$K(x,y) := xy \quad f(x) := 2x \quad ut(x) := 3x$

$a := 0 \quad b := 1 \quad n := 6 \quad h := (b-a)/(n-1) \quad i := 1..n \quad x_i := a + (i-1)h$

$f_i := f(x_i) \quad ut_i := ut(x_i) \quad \varphi(i,x) := x^{i-1} \quad j := 1..n$

$\alpha_{i,j} := \int_a^b \varphi(i,x)\varphi(j,x)dx \quad \beta_{i,j} := \int_a^b (\int_a^b K(x,y)\varphi(j,y)dy)\varphi(i,x)dx \quad ff_i := \int_a^b \psi(i,x)f(x)dx$

$i := 1..n \quad j := 1..n \quad M_{i,j} := \alpha_{i,j} - \beta_{i,j} \quad c := lsolve(M, ff)$

$c^T = (-0\ 3\ 0.505\ \ 0.206\ -0.206\ 0.206)$

$$u(x) := f(x) + \sum_{j=1}^{n} c_j \varphi(j,x) \to \frac{1848x^5}{8989} - \frac{1848x^4}{8989} + \frac{2604x^3}{8989} + \frac{1848x^2}{8989} + \frac{22514x}{8989} - \frac{1}{8989}$$

$ut(x)^T = (0\ 0.6\ 1.2\ 1.8\ 2.4\ 3) \quad u(x)^T = (-0\ 0.511\ 1.05\ 1.629\ 2.267\ 3)$

Individual topshiriqlar

Quyidagi integral tenglamalar turli xil usullar bilan echilsin.

13.4.17-jadval

N	Integral tenglama	Javoblar
1	$y(x) - \int_{0}^{1} e^{xs} y(s)ds = x - x^{-1}e^{-x} - x^{-2}(e^{x} - 1)$	
2	$y(x) - 0.5\int_0^1 xsy(s)ds = 5x/6$	
3	$y(x) - 6\int_0^{\pi/3} \sin^2(x)y(s)ds = 3x - 10x/11$	
4	$y(x) + \int_0^x e^{x-s}y(s)ds = e^x$	
5	$\int_0^x \exp(2(x-s))y(s)ds = \sin(x)$	
6	$u(x) - \int_0^1 e^{\alpha x - t}u(t)dt = e^x - e^{\alpha x}$	
7	$u(x) - \int_0^1 e^{-\alpha x - t}u(t)dt = e^x - e^{-\alpha x}$	
8	$u(x) - \int_0^1 \sin(\alpha x t)u(t)/t\,dt = f(x), (f = x, \sqrt{x}, x^2)$	
9	$u(x) - \int_0^1 (1+s)(e^{xs} - 1)u(s)ds = f(x), (f = 1/x, 1-x, e^{-x})$	

Content

QAYDLAR UCHUN

QAYDLAR UCHUN

10	$u(x)-\int_0^1 \cos(axt)u(t)/t = f(x),(f = x, \sqrt{x}, x^2)$	
11	$u(x)-\int_{-\pi}^{\pi} u(s)/(6.8-3.2\cos(x+s)ds = 25-16\sin(x^2)$	
12	$u(x)-\int_0^1 u(s)/(1+x^2+s^2)ds = 1.5-x^2$	
13	$y(x)-\int_0^{0.5}(1+s))y(s)/(2+\sin\pi(x+s))ds = 1+\sin(x)$	
14	$y(x)-\int_0^{0.96}(1+x+s)y(s)/(2+x^2+s^2)ds = e^{-x}$	
15	$y(x)-\int_0^1(1+x+s)y(s)/(2+x+s)ds = 1-x^2$	
16	$y(x)-4\int_0^1 xsy(s)ds = 1$	
17	$y(x)-3\int_0^2 xsy(s)ds = 3x-2$	
18	$y(x)-4\int_0^{\pi/2}\sin(x)\cos(s)y(s)ds = \sin(x)$	
19	$y(x)-\int_0^x \exp(-2(x-s))y(s)ds = 1+x$	
20	$\int_0^x \exp(x-s)y(s)ds = sh(x)$	
21	$u(x)-\int_0^x e^{\alpha x-t}u(t)dt = e^x - xe^{\alpha x}$	
22	$u(x)-\int_0^x e^{-\alpha x-t}u(t)dt = e^x - xe^{-\alpha x}$	
23	$u(x)-\int_0^1 xsu(t)/\sqrt{1+\alpha xs}ds = f(x),(f = 1+x, e^{-x}, \sqrt{x})$	
24	$u(x)-\int_0^1(1+s)(e^{\alpha x}-1)u(s)ds = f(x),(f = 1/x, 1+x, e^{-x})$	
25	$u(x)-\int_0^1 \cos(\alpha xt)u(t)dt = f(x),(f = x, \sqrt{x}, x^2)$	
26	$y(x)-\int_0^x e^{-x-t}y(s)ds = 0.5(e^{-x}+e^{-3x}),[0,1]$	
27	$y(x)-0.5\int_0^x y(s)/(2-\sin\pi x)ds = 2-\sin(\pi x)$	
28	$y(x)-\int_0^x y(s)/(1+x+s)ds = 1+x$	
29	$y(x)-\int_0^x y(s)/(1+e^{-xs})ds = ch(x)$	

30	$y(x)-\int_0^1 sh(xs)y(s)ds=1-x^2$	

$$1.u(x)-\int_0^1 e^{\alpha x-t}u(t)dt=e^x-e^{\alpha x},\bar{u}(x)=e^x,\quad 2.u(x)-\int_0^1 e^{-\alpha x-t}u(t)dt=e^x-e^{-\alpha x},\bar{u}(x)=e^x,$$

$$3.u(x)-\int_0^x e^{\alpha x-t}u(t)dt=e^x-xe^{\alpha x},\bar{u}(x)=e^x,\quad 4.u(x)-\int_0^x e^{-\alpha x-t}u(t)dt=e^x-xe^{-\alpha x},\bar{u}(x)=e^x.$$

Nazariy savollar va topshiriqlar ?

1.Integral tenglama nima, uning yechimi nima?

2. Integral tenglama necha tur bo'ladi, qaysilar?

3. Voltyer 1 va 2 tur integral tenglamasi nima ?

4. Fredgolm 1 va 2 tur integral tenglamasi nima ?

5. Fredgolm 1 va 2 tur integral tenglamasining nokorrektligi?

Ko'p uchraydigan asosiy qisqartmalar:

ODT-oddiy differentsial tenglama,

XHDT-xususiy hosilali differentsial tenglama

PDT-parabolik differentsial tenglama

GDT-giperbolik differentsial tenglama

EDT-elliptik differentsial tenglama

ChATS-chiziqli algebraik tenglamalar sistemasi

ChAS -chekli ayirmali sxema

AS- ayirmali sxema

NT-nochiziq tenglama

NTS-nochiziq tenglamalar sistemasi

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